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Mathematical Analysis of Business Cycles - A Reduced-form Model

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This paper aims to develop a mathematically reduced-form model to analyse and interpret business cycle fluctuations and their consequences on economic stability. The proposed model utilises national income identity and a set of six behavioural equations including consumption, investment, taxes, money demand, exports and imports. Using the tools of matrix algebra, a (7×7) matrix is constructed from the constant term and co-efficients of six independent variables. The model's stability relative to the changes in the price level, exchange rate, international trade, fiscal and monetary policies, is viewed from eigenvalues, eigenvectors, and eigenspace generated from the model matrix. Coupled with its described protocol, the proposed reduced-form model has proved competency in connoting different theories of the business cycle, addressing growth issues, and shaping appropriate economic policies.

Keywords: Business cycle, Reduced-form, Matrix stability, Economic stability, National income

JEL: B41; C13; C62; E10; E32; E63; N10; O40

Introduction:

The concept of the business cycle, also known as the economic or trade cycle, refers to consistent patterns in fluctuations of macroeconomic variables, including output, consumption, investment, employment, prices and interest rates, and their influence on aggregate economic activity.

The most renowned definition of the business cycle was provided by Burns and Mitchell (1946, p.3): "*Business cycles are a type of fluctuations found in the aggregate economic activity of nations that organise their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in numerous economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle. This sequence of changes is recurrent but not periodic in duration, business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter* *cycles of similar character with amplitudes approximating their own.*" (Burns, A.F., and Mitchell W.C, 1946)

There are several explanations for the business cycle phenomenon, which include the link of the business cycle to the proposition of money and investment. In this case, the business cycle is interpreted as a product of an imbalance in the money and credit markets or in the supply of money and the demand for investment. In case of an imbalance of money and credit market, increasing bank credit stimulates production by increasing profits and thus income, as a result, high credit increases investment. Disequilibrium occurs when actual and desirable investment compared to voluntary savings are not balanced, or when actual investment does not match consumption. In case of imbalance in the supply of money and the demand for investment, the economy shrinks as a result of the reduction of bank credit when deposits are reduced because there are better investment opportunities or in the case of investment when the actual investment is lower than desired for the stability of voluntary savings for financing investments, in which case profits and purchasing power decrease, allowing for a recession. The previous approach fails to explain the impact of interest rate and non-monetary factors on business cycles. Demand for new investments, cost structure, business expectations, market volatility, as well as a focus on investment and consumption may not provide a reasonable justification that explains economic imbalances (Block, & Garschina, 2009).

There is an approach to business cycles linking investment to new inventions in industry and business Organisations informing changing production methods and the introduction of new concepts in the cost-cutting process. New inventions increase investment and thus stimulate credit, income and profits, in addition, it reduces the unemployment rate to zero. When inventions are discouraged, prices begin to fall, leading to the loss of creditors and the emergence of recession (Schumpeter). This interpretation may not be sufficient to contain variables other than the relationship between investment and invention in explaining the expansion and contraction of the economy, keeping in mind international trade relations and their role in affecting international economic conditions (Schumpeter, J. A, 1939).

As a result of the great depression, Keynes recognised the importance of aggregate demand as a key factor in interpreting economic variables and business cycles. The classical interpretation failed to restore balance through the invisible hand and market mechanism. From Keynes' point of view, stimulating demand leads to higher profits exceeding the market interest rate and hence motivates investment and improves marginal capital efficiency as a result of increased inventions and the effect of the multiplier principle. On the other hand, lower marginal capital efficiency below the market interest rate discourages investment, credit and shrinks economic activity. This theory takes into account the role of the multiplier when measuring the investment effect while ignoring the role of the accelerator (Zarnowitz, V, 1984).

The accelerator effect persuades Samuelson to bring it alongside the multiplier to interpret economic cycles through their effect in conjunction with autonomous and induced investment, the latter resulting from dependent factors such as the entry of a new product production method or market increasing income and investment in capital goods through the principle of the multiplier, while increased demand for consumer goods is generated by the principle of the accelerator. These two processes are used to interpret business cycles with the so-called multiplier - accelerator interaction, attempting to interpret the causes of fluctuations in income and thus imbalance, assuming the absence of government and external trade, which simplifies the model and ignores other factors contributing to imbalances such as business expectations and changes in the structure of consumer taste and preference (Samuelson, 1939).

In previous approaches, investment has been seen as an important element in the occurrence of economic imbalances. Hickes developed his theory by introducing relationships between investment, savings and the multiplier principle which are mentioned in previous theories such as Clark's accelerator principle, Samuelson's multiplier-accelerator model as well as Harrod-Domar's growth theory. Hicks assumed that equilibrium occurs when the actual growth rate is equal to the natural growth rate. The natural growth rate is determined by investment and voluntary savings, considering that warranted growth resulted from the multiplier and accelerator effects and the natural growth rates of population, in addition to capital and technological rates of growth. The focus is always on the gap between wage and non-wage income according to which lag consumption is determined, while investment which is used to replace capital goods is a function of current income. Overlapping all these factors creates an imbalance between actual and natural growth rates, leading to inflation or recession, whereas economic stability occurs as a result of adjusting these variables in a way that leads to the actual growth rates being equal to natural growth rates. Investment savings relationships can be adjusted to avoid fluctuations in economic activity; therefore, the business cycle is expected to occur when resources are scarce, leading to economic recession which will continue until the accelerated principle starts to work. Hicks assumed the constancy of multiplier accelerator principles and the linearity of the consumption function, while in actual economic practices shows the opposite, therefore a lack of interpretation of business cycles exists when considering only the abstract view of the phenomenon (Hicks, J. R, 1989).

This study develops a mathematically reduced-form model to interpret and analyse business cycle fluctuations and their consequences on economic stability, as well as their implications on aggregate economic activity. The study uses the methodology of mathematical modelling and employs the tools of matrix algebra, the reduced-form model's characteristics and stability are identified from eigenvalues, eigenvectors and eigenspace.

The study assumes the interdependence and interaction between all economic sectors in producing and formulating business cycles as well as determining their paths and size.

The importance of the study stems from the fact that it combines most of the theories that explain business cycles, as the proposed model captures economic variables that determine the size, direction, and time- elapse of the business cycle, and traces their final effect on economic sectors, therefore the study attempt to make a theoretical contribution to existing literature by providing an approach to business cycles emphasising the economic implication of a considerable number of variables included in the proposed reduced-form model.

Literature Review:

All economic systems strive to achieve a set of broad economic goals, chief among them economic stability. Although the state of full employment varies from one economy to another depending on size, potential resources and demographic characteristics, the degree of economic stability is observed by a minimum rate of inflation and natural rate unemployment. Demographic variables are included as they play a key role in growth theories which target economic stability.

Economic stability can be studied by analysing the behaviour of the business cycle using a package or matrix of economic variables that cover minimum macro variables affecting indicators of economic stability, such as inflation, unemployment or imbalance in the balance of trade. It is therefore necessary for the matrix used to study economic stability, to be as comprehensive as possible containing all economic variables pointed towards different theories and approaches studying the business cycle phenomenon.

Blundell, (2017) focuses on a structurally reduced-form model where the structure of decision making is incorporated in the model specification. His paper identifies three distinct related objects, which are structurally “deep” parameters, underlying mechanisms and policy counterfactuals. The focus of his research enables a good understanding of the mechanisms underlying observed behaviour and that provide significant insights about policy counterfactuals.

Separating the standard incomplete-markets model of consumption and inequality with the new Keynesian model of nominal rigidities and business cycles, McKay & Reis, (2016) measured the effect on the dynamics of the business cycle. The model includes most of the main potential stabilisers in US data and the theoretical channels by which they may work. Their findings show that stabilisers have a significant role when monetary policy is subjected to zero lower bound and significantly affect welfare via the provision of social insurance.

To identify international tendencies amongst credit and output cycles, Kurowski, & Rogowicz, (2018) assessed the synchronisation of business and financial cycles at internal and international levels using a long-term series spanning including multiple countries accounting for almost 60% of global gross domestic product. Their study performed cycle synchronisation calculations based on wavelet analysis. Their findings confirmed that the pre-crisis period was one of internal synchronisation between financial and output cycles across countries, while the post-crisis period has been marked by international synchronisation, moreover, they confirmed the existence of one global financial cycle.

Rigatos, et. al. (2019) suggest a new non-linear optimal control method for the stabilisation of the business cycles of interrelated finance agents. Their model undergoes approximate linearisation around a temporary operating point, and according to Taylor’s series expansion of the dynamic model, the computation of Jacobian matrices, and Lyapunov’s stability analysis, the control scheme is globally and asymptotically stable.

Applying a non-parametric method to gauge the degree of synchronisation between credit and financial stability, Koong, Law, & Ibrahim, (2017) examine the degree of synchronisation between credit expansion and financial stability in Malaysia at aggregated and disaggregated levels. Their findings indicate negative synchronisation between business credit and financial stability, indicating that an expansion in business credit would lead to financial instability. In addition, they confirm that increasing household credit has no negative influence on Malaysian financial stability.

The reviewed literature is part of a number of diversified studies exploring the business cycle and its implications on economic activity.

The Model & its Protocol:

The model consists of national income identity and the behavioural equations of its components, which include national income, consumption, investment, tax, demand for money, exports and imports. These components constitute the dependent (endogenous) variables of the model, while the set of independent (exogenous) variables consist of indirect tax, government expenditure, money supply, exchange rate, price level and lag of income. The mathematical model is transformed into a reduced-form in which each dependent variable is formulated from the constant term and independent variable.

In transforming behavioural equations (income, consumption, tax, investment, demand for money, exports and imports) interest rate is expressed in terms of the money market equilibrium to be included in the identity equation, bearing in mind that interest rate is a function of the price level in demand for money equation in reduced-form.

The procedures of the model protocol are:

First, co-efficients of the mathematical model are estimated using one of the simultaneous equations' estimators.

Second, estimated co-efficients must be filled into reduced-form equations.

Third, constituting a (7×7) matrix from the constant term and co-efficients of the six independent variables in the seven behavioural equations.

Fourth, subtracting the matrix form, the identity matrix multiplied by Lamda (λ) and equating the determinant by zero to form a singular matrix.

Fifth, evaluating the properties of the characteristic matrix by examining eigenvalues, eigenvectors and eigenspace.

The stability of the model is tested and evaluated according to the above-mentioned procedures.

The Model:

$$Y = C + I + G + (X + C_{inf}) - (M + C_{outf}) \quad (1)$$

(Y≡ National Income; C ≡ Consumption; I ≡ Investment; G ≡ Government Expenditure; X ≡ Exports; C_{inf} ≡ Capital Inflow; M ≡ Imports; C_{outf} ≡ Capital outflow)

$$C = \alpha + \beta Y_d + \Delta Y_{t-1} \quad (2)$$

(C ≡ Consumption; $\alpha \equiv$ constant ; $Y_d \equiv$ Disposal Income;
$\beta \equiv$ marginal propensity to consume; Y_{t-1} ≡ lag of income; $\Delta \equiv long\ run\ growth\ rate$)

$$I = d - ei + vy \quad (3)$$

(I ≡ Investment; $d \equiv constant$; $i \equiv$ interest rate; $e \equiv$ elasticity of investment with respect to interest rate; $y \equiv National\ income$; $v \equiv accelator\ coefficient$)

$$T = t_0 + tY \quad (4)$$

(T ≡ tax; $t_0 \equiv fixed\ tax$; $Y \equiv$ national income; t ≡ $tax\ rate$)

$$M^d = f - hi + ky \quad (5)$$

($M^d \equiv demand\ for\ money$; $f \equiv$ constant; i ≡ interest rate; h ≡ elasticity of money supply with respect to interest; y ≡ $national\ income$; k ≡ proportion of money in hand)

$$M^d = M^s \quad (6)$$

($M^s \equiv$ money supply)

$$X = \sigma - \delta P + \epsilon Y - \phi E \quad (7)$$

(X ≡ exports; $\sigma \equiv constant$; $P \equiv price\ level$; $\delta \equiv$ elasticity of exports with respect to price level; Y ≡ national income; $\epsilon \equiv$ elasticity of exports with respect to income; E ≡ exchange rate; $\phi \equiv$ elasticity of exports with respect to exchange rate)

$$M = \theta - \rho P + \mu Y - \chi E \quad (8)$$

(M ≡ $Import$; $\theta \equiv constant$; $P \equiv price\ level$; $\rho \equiv$ elasticity of imports with respect to price level; Y ≡ national income; $\mu \equiv$ elasticity of imports with respect to income; E ≡ exchange rate; $\chi \equiv$ elasticity of imports with respect to exchange rate)

$$C_{inf} = m + \Psi i \quad (9)$$

(C_{inf} ≡ Capital inflow; m ≡ constant; I ≡ interest rate; Ψ ≡ elasticity of capital inflow with respect to interest rate)

$$C_{outf} = n - \varpi i \quad (10)$$

(C_{outf} ≡ Capital inflow; n ≡ constant; I ≡ Interest rate; ϖ ≡ elasticity of capital outflow with respect to interest rate).

The Reduced-Form:

The reduced-form model is constructed from the seven behavioural equations of dependent variables which include (income, consumption, tax, investment, demand for money, exports and imports). Each equation is expressed in term of constant and six independent variables such as (indirect tax, government expenditure, money supply, price level, exchange rate and lag of income) as follows:

$$y = (\alpha + d + m - \theta - n + \sigma) + [\beta - \beta t + vy + \epsilon - \mu]y - \beta t_0 + G - (e - \Psi - \varpi)i - (\partial + \rho)p - (\varphi + x)E + \Delta y_{t-1}$$

$$y = (\alpha + d + m - \theta - n + \sigma) + [\beta - \beta t + v + \epsilon - \mu]y - \beta t_0 + G - (e - \Psi - \varpi)\left(\frac{f}{h} + \frac{k}{h}y - \frac{M^s}{h}\right) - (\partial + \rho)p - (\varphi + \chi)E + \Delta y_{t-1}$$

$$y = (\alpha + d + m - \theta - n + \sigma) + [\beta - \beta t + v + \epsilon - \mu]y - \beta t_0 + G - (e - \Psi - \varpi)\frac{f}{h} - (e - \Psi - \varpi)\frac{k}{h}y + \frac{(e - \Psi - \varpi)}{h}M^s - (\partial + \rho)p - (\varphi + \chi)E + \Delta y_{t-1}$$

$$y = \left[\alpha + d + m - \theta - n + \sigma - \frac{ef}{h} - \frac{\Psi f}{h} + \frac{\varpi f}{h}\right] + \left[\beta - \beta t + v + \epsilon - \mu - \frac{ek}{h} + \frac{\Psi k}{h} + \frac{\varpi k}{h}\right]y - \beta t_0 + G + \frac{(e - \Psi - \varpi)}{h}M^s - (\partial + \rho)p - (\varphi + \chi)E + \Delta y_{t-1}$$

$$y\left[1 - \beta + \beta t - v - \epsilon + m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right] = \left[\alpha + d + m - \theta - n + \sigma - \frac{ef}{h} - \frac{\Psi f}{h} + \frac{\varpi f}{h}\right] - \beta t_0 + G + \frac{(e - \Psi - \varpi)}{h}M^s - (\partial + \rho)p - (\varphi + \chi)E + \Delta y_{t-1}$$

1- National income reduced-form equation:

$$y = \frac{(\alpha + d + m - \theta - n + \sigma - \frac{ef}{h} + \frac{\Psi f}{h} + \frac{\varpi f}{h})}{(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h})} - \left[\frac{\beta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0$$

$$+ \left(\frac{I}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right) G$$

$$+ \left[\frac{\frac{e}{h} - \frac{\Psi}{h} - \frac{\varpi}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] M^s$$

$$- \left(\frac{(\partial + \rho)}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right) P$$

$$- \left(\frac{\varphi + x}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right) E$$

$$+ \left(\frac{\Delta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right) y_{t-1}$$

2- Consumption reduced-form equation:

$$C = \left[\alpha + \left[\frac{\left(\alpha + d + m - \theta - n + \sigma - \frac{ef}{h} + \frac{\Psi f}{h} + \frac{\varpi f}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}\right)}\right] \beta\right]$$

$$- \left[\frac{\beta + \beta^2 t - \beta v - \beta\epsilon - \beta m + \frac{\beta ek}{h} - \frac{\beta \Psi k}{h} - \frac{\beta \varpi k}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] t_0$$

$$+ \left[\frac{\beta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] G$$

$$+ \left[\frac{\frac{\beta e}{h} - \frac{\beta \Psi}{h} - \frac{\beta \varpi}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] M^s$$

$$- \left[\frac{\beta\partial + \beta\rho}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] P$$

$$- \left[\frac{\beta\varphi + \beta\chi}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] E$$

$$+ \left[\frac{\Delta + \Delta\beta t - \Delta\epsilon - \Delta m + \frac{(\Delta h + ek)}{h} - \frac{(\Delta h + \Psi h)}{h} - \frac{(\Delta h + \varpi h)}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}}\right] y_{t-1}$$

3- Tax reduced-form equation:

$$T = \left[\frac{\left(t\alpha + td - tm - t\theta - tn - t\sigma - \frac{tef}{h} + \frac{t\Psi f}{h} + \frac{t\varpi f}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right)}\right] - \left[\frac{\beta - I + v + \epsilon - m - \frac{ek}{h} + \frac{\Psi k}{h} + \frac{\varpi k}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0$$

$$+ \left[\frac{t}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] G$$

$$+ \left[\frac{\left(\frac{te}{h} - \frac{t\Psi}{h} - \frac{t\varpi}{h}\right)}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] M^s$$

$$- \left[\frac{t\partial + t\rho}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] P$$

$$- \left[\frac{t\varphi + tx}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] E$$

$$+ \left[\frac{t\Delta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] y_{t-1}$$

4- Investment reduced-form equation:

$$I = d - \frac{ef}{h} - \frac{ekg}{h} + \left[\frac{\left(v\alpha + vd - vm - v\theta - vn + v\sigma - \frac{vef}{h} + \frac{v\Psi f}{h} + \frac{v\varpi f}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right)}\right]$$

$$- \left[\frac{v\beta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0$$

$$+ \left[\frac{v}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] G$$

$$+ \left[\frac{e}{h}\left(\frac{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} + \frac{\varpi k}{h}\right) + \left(\frac{ve}{h} - \frac{v\Psi}{h} - \frac{v\varpi}{h}\right.}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right]M^s$$

$$- \left[\frac{v\partial + v\rho}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] P$$

$$- \left[\frac{v\varphi + v\chi}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] E$$

$$+ \left[\frac{v\Delta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] y_{t-1}$$

5- Demand for Money reduced-form equation:

$$M^d = f - hz + gp + \left[\frac{\left(k\alpha + kd - km - k\theta - kn + k\sigma - \frac{kef}{h} + \frac{\Psi k^2}{h} + \frac{\varpi k^2}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right)}\right]$$
$$- \left[\frac{k\beta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0$$
$$+ \left[\frac{k}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] G$$
$$+ \left[\frac{\left(\frac{ke}{h} - \frac{k\varphi}{h} - \frac{k\varpi}{h}\right)}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] M^s$$
$$- \left[k\partial + k\rho + q - q\beta t - qv - q\epsilon - qn + \frac{qek}{h} - \frac{q\Psi k}{h} - \frac{q\varpi k}{h}\right] P$$
$$- \left[\frac{k\varphi + k\chi}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] E$$
$$+ \left[\frac{\Delta k}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] y_{t-1}$$

Note: $i = z - qP$, *Where:* I ≡ *interest rate*; z ≡ *constant*; P ≡*price level*; q ≡ *elasticity of interest rate with respect to price level*

6- Export reduced-form equation:

$$X = \left[\frac{\left(\sigma + \epsilon\alpha + \epsilon d + \epsilon m - \epsilon\theta - \epsilon n - \epsilon\sigma - \frac{\epsilon ef}{h} + \frac{\epsilon\Psi f}{h} + \frac{\epsilon\varpi f}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right)}\right]$$
$$- \left[\frac{\epsilon P}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0$$
$$+ \left[\frac{\epsilon}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] G$$
$$+ \left[\frac{\left(\frac{\epsilon e}{h} - \frac{\epsilon\Psi}{h} - \frac{\epsilon\varpi}{h}\right)}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] M^s$$
$$- \left[\frac{\left(\partial - \partial\beta + \partial\beta t - \partial v - \partial\epsilon - \partial G + \partial m + \frac{\partial ek}{h} - \frac{\partial\Psi}{h} - \frac{\partial\varpi}{h}\right) + \epsilon\partial - \epsilon\rho}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] P$$
$$- \left[\frac{\phi + \phi\beta + \phi\beta t - \phi v - \phi\epsilon - \phi m + \frac{\phi ek}{h} - \frac{\phi\Psi k}{h} - \frac{\phi\varpi k}{h} + \epsilon\phi - \epsilon x}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] E$$
$$+ \left[\frac{\epsilon\Delta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] y_{t-1}$$

7- Import reduced-form equation:

$$
\begin{aligned}
M &= \left[\frac{\left(\theta - \theta\beta + \theta\beta t - \theta v - \theta\epsilon - \theta m + \frac{\theta e k}{h} - \frac{\theta \Psi k}{h} + \frac{\theta \varpi k}{h} + \mu\alpha + \mu d - \rho m - \mu\theta + \mu\sigma - \frac{\mu e f}{h} + \frac{\mu \Psi f}{h} + \frac{\mu \varpi f}{h}\right)}{\left(1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}\right)}\right] \\
&- \left[\frac{\mu\beta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] t_0 + \left[\frac{\mu}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] G \\
&+ \left[(\frac{\frac{\mu e}{h} - \frac{\mu \Psi}{h} - \frac{\mu \varpi}{h}}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] M^s \\
&- \left[\frac{\rho - \rho\beta + \rho\beta t - \rho v - \rho\epsilon - \rho\mu + \frac{\rho e k}{h} - \frac{\rho \Psi k}{h} - \frac{\rho \varpi k}{h} - \mu\partial - \mu\rho}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] P \\
&- \left[\frac{x - x\beta + x\beta t - xv - x\epsilon - x\mu + \frac{x e k}{h} - \frac{x \Psi k}{h} - \frac{x \varpi k}{h} - \mu\phi - \mu x}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] E \\
&+ \left[\frac{\mu\Delta}{1 - \beta + \beta t - v - \epsilon - m + \frac{ek}{h} - \frac{\Psi k}{h} - \frac{\varpi k}{h}}\right] y_{t-1}
\end{aligned}
$$

Results and Discussion:

Using the above reduced-form structural equations, a 7×7 matrix is constructed from the constant term and the co-efficients of six independent variables, for the analysis of the business cycle we examine matrix stability which reflects economic stability, resulting in the following:

1- Four states of stability can be determined from the magnitude and signs of eigenvalues: momentically stable, when the values are negative and less than one, oscillated stable when the values are negative and greater than one, momentically unstable when values are positive and less than one, oscillated unstable when the values are positive and greater than one.
   The eigenvectors determine the direction of the cycle. A positive direction means the cycle state will diverge from the equilibrium, while a negative direction means the state of the cycle converges to equilibrium.
   The eigenspace specifies cycle's stability domain by determining the explanatory variables which cause the fluctuations of the cycle.
2- The constant term in the matrix counts for variables such as technology, knowledge, and administration, which contribute the stability not included in the matrix. The extent to which these variables contribute to the state of stability is imbedded in the magnitude of the constant term and its relations to eigenvalues, eigenvectors and eigenspace. The knowledge economy extends the long-run stability of the cycle and reverses its direction when non – stability exists.

3- The co-efficients of government expenditure and tax reflect the impact of the fiscal policy regarding the stability of the cycle, while the co-efficient of money supply reflects the role of monetary policy. Accordingly, policymakers specify the appropriate type of policy that accelerates stability, reversing the cycle, or preserving the current state of stability. The choice between fiscal or monetary policies is therefore a reflection of the matrix's eigenvalues, eigenvectors and eigenspace.

4- The matrix's price level and exchange rate co-efficients show the economic degree of sensitivity to changes in domestic price level and international trade. This reflects the stability response of the economy to internal and external shocks, furthermore, the co-efficients of lag income show the interaction of the long-run factors affecting stability with short-run ones in a manner specifying the existence and stability of that equilibrium.

5- On the one hand, the demand and supply for money co-efficients in the matrix explain the degree of these variables affecting the state of stability of a given economy, while on the other hand their interaction with the rest of variables in generating stability or instability, as well as investment along with money supply determine the power of such variables in explaining the business cycle (Cooper, Haltiwanger & Power, 1999).

6- Since the matrix system mostly comprises of the co-efficients of demand-side variables, this could strengthen the hypothesis that the demand side has a leading role in directing the economy. On the other hand, Price level co-efficients in the matrix count for the role of the supply side.

7- The co-efficient of exchange rate determines the way of structuring the allocation of resources in addition to motivating the entrepreneur, keeping in mind the existence of the exchange rate in shaping the business cycle.

8- The components of multipliers and accelerators are have a significant representation in the matrix, indicating the governing role of multiplier accelerator interaction theory in explaining the business cycle. moreover, the stability of a given economy can be traced according to the volume of multiplier and accelerator. This assures an extended phase of stability if the accelerator co-efficient is smaller than the multiplier co-efficient, which may extend the phase of stability and quickly reverse the cycle towards instability, otherwise, the opposite can happen if the accelerator co-efficient is greater than the multiplier co-efficient.

9- The warranted rate of growth is embodied in the matrix and is affected by the indigenous factors of growth which depend on the degree of the knowledge economy, along with exogenous factors such as investment and saving. This growth pattern is coupled with golden age; natural growth patterns of growth imitate the Harrod–Domar's vision towards the interpretation of business cycles in relation to growth.

The degree of stability or instability can be detected from the volume of the warranted compared to the actual rate of growth. When the economy reduces naturally, according to exogenous or endogenous factors, warranted or golden age rate of growth the economy can easily enjoy a long period of stability by the of law of the actual rate of growth.

Concluding Remarks:

- The reduced-form model reveals a consistency in representing the proportion contribution of all variables in the model to instability in terms of their co-efficients in the matrix.
- The reduced-form model has proved competency in connoting various theories of the business cycle and their effectiveness derived from matrix co-efficients.
- The properties of the reduced-form matrix facilitate shaping appropriate economic policy targeting economic stability, as well as growth issues such as natural, warranted, or golden age rates of growth and their implication regarding stability.
- Finally, the suggested model and its protocol can make a positive contribution to the existing literature regarding business cycle interpretation and monitoring.

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