



The Impact of Lesson Study on Achievement in Mathematical Problem Solving and Higher Order Thinking Skills (HOTS) among Foundation Level Students

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Problem solving and higher order thinking skills (HOTS) have always been emphasised in most mathematics curriculums. However, students in general have not shown significant improvement, as has been evidenced in international assessments in mathematics. This quasi experimental research was conducted to study the impact of the implementation of a lesson study on achievement in mathematical problem solving and HOTS, among students of a Foundation Centre in Malaysia. A comparison between the performances of genders was also explored. Foundation Centres offer pre-university programmes, bridging students from high school to their undergraduate degrees. The lesson study involved eight lecturers and 45 and 50 students respectively in the control and experimental groups. During the lesson study sessions and the research lesson process, the mathematics lecturers emphasised upon mathematical problem solving and HOTS through individual and teamwork activities. The research lessons were taught in the experimental class through a student-centred approach with an emphasis on problem solving and HOTS, whereas in the control group, the traditional method was conducted through a lecturer-centred approach with an emphasis upon the solving of exercises. The students' abilities in mathematical problem solving and HOTS were examined through five tests on the topic of mathematics function. An independent t-test, ANOVA and repeated measures one-way ANOVA were conducted for analysis. The results of this study showed that the implementation of the



lesson study approach for the experimental group increased the students' skills in problem solving and HOTS significantly, while in control group, no significant differences were established. Furthermore, no differences in gender were established, which is a good sign for the education system.

Keywords: *Lesson Study, Research Lesson, Mathematical Problem Solving, Students' Achievement, Higher order thinking.*

Introduction

The teaching of mathematics, especially at the upper level of secondary schools, university preparatory programmes and higher education, is challenging since the contents and problems are more complex in comparison to the lower school levels. In Malaysia, all students who completed and passed secondary school must undergo a university preparatory programme which are offered through several pathways; foundation, matriculation, A-level or Form Upper Six in secondary school. A pre-university programme ranges between 12 to 18 months and the eligibility depends on the students' performance in their Secondary School Examination. On the other hand, a student's selection to universities and the study programme depends entirely on their grade point average obtained in the pre-university programmes. The courses in the pre-university programmes are mainly mathematics, the sciences (Physics, Chemistry, Biology), English and Information Communication Technology. Thus, doing well for the pre-university programme is of the utmost importance to the students. In this context, the teaching competency of the lecturers at the pre-university level is also critical in ensuring that the students, who are the output of secondary schools, improve tremendously to secure placement in good study programmes and universities. Thus, the lecturers must have the content knowledge and pedagogical content knowledge (PCK) to ensure that their students will achieve good grades. Several years ago, it was not necessary for the lecturers in the pre-university programmes to have a diploma in education for teaching certification. In recent years, they have been encouraged to undergo the post graduate diploma in education. Nevertheless, having a diploma in education is not an assurance that the lecturers have adequate knowledge on how to teach. Every new topic may need a certain pedagogical approach and certain insights on how best to deliver it. Thus, lecturers should continuously strive to improve their knowledge. One of the ways to continually upgrade one's content knowledge and PCK is to learn and share from each other on the best practices to teach a certain content to a specific group of students. As highlighted by Fujii (2016), and Mon, Dali, and Sam (2016), collaborative work can improve educators knowledge on teaching, especially their content knowledge and PCK, in many ways.



Japanese teachers have been using the lesson study approach since the nineteen-fifties (Abiko, 2011) as part of teacher professional development. Yoshida (1999) translated the Japanese term “*Jugyo Kenkyu*” to lesson study and it was a hot topic during the last two decades among researchers and educators (Kazemi, Zaman, & Ghafar, 2014). Teachers are actively involved in both the process and the products with a focus on the content and specifically, on students learning this content (Coenders & Verhoef, 2019; Penuel, Yamaguchi, Gallagher, & Fishman, 2007). It requires teachers to collaboratively work on a mathematics topic and spend a lot of time in planning the lesson, teaching and/or observing the lesson, and reflecting and discussing the taught lesson in order to improve students’ achievements in learning and problem solving (Matanluk, Johari, & Matanluk, 2013). These lessons, “*gakushushido-an*”, in Japanese are called research lessons (Fujii, 2016) or study lessons (Fernandez & Yoshida, 2004). A lesson study points to a cycle of pedagogical progress of which the research lesson is the key component (McDonald, 2009).

The purpose of the lesson study approach is to improve the quality of teaching through “reflexive, recursive and collaborative” processes (Dudley, 2011). Teachers need to improve their content knowledge and PCK continuously to enhance their students’ abilities in mathematical problem solving and HOTS. The purpose of this experimental research was to determine the impact of mathematics lecturers’ professional development through a lesson study on achievement in problem solving and HOTS among students who were undergoing the foundation level. A foundation programme is one of the pre-university programmes in Malaysia and is conducted by a university.

In conducting a lesson study, Fujii (2014) suggested five phases for the implementation, as follows:

- a. Goal Setting: Teachers focus on long-term goals in order to improve the students’ learning, problem solving and achievement.
- b. Lesson Planning: Teachers collaboratively design a research lesson with suitable materials to improve students’ abilities.
- c. Research Lesson: One member of lesson study group teaches the research lesson and other members observe and collect data in order to improve the lesson.
- d. Post-lesson Discussion: Through a post-lesson discussion, teachers consider students’ learning, students’ misunderstanding, different solutions for problems, unit design and disciplinary content to enrich the lesson.
- e. Reflection: Teachers discuss the new questions for the lesson and they collaboratively plan to solve these problems in the next cycle of the lesson study. In this phase, the mathematics teachers also prepare a report about the research lesson.

Mathematical Problem Solving

According to the National Council of Teachers of Mathematics (NCTM) (2000), a mathematics task is considered a problem if students are engaged with the task for the first time and the task is challenging to the students. Otherwise, it is merely a mathematics exercise. Therefore, mathematical problem solving refers to students being engaged in a task that they have not encountered before. The distinction between what is considered a problem and an exercise depends on many factors, including the grade level, mathematics competence, learning materials, the way it was taught, and the time given to complete the task. For instance, the following mathematics problem will be considered a mathematics exercise if it has been discussed in class before:

Find the range of the function $f(x) = \frac{x^2}{x^2+1}$.

However, if the lecturer injected some changes in this mathematics exercise, students will then be provided with a new challenge and this will become a mathematics problem:

Find the range of the functions $g(x) = \frac{5x^2+4}{1+x^2}$, $h(x) = \frac{3x^4}{1+x^4}$ and $k(x) = \frac{7x^2}{1+x^4}$.

Polya (1945) suggested four phases for mathematical problem solving: namely, understanding the problem, planning a strategy, performing the plan, and confirming the answer. Since then, other models with a different number of steps and phases have been introduced by mathematics educators and researchers. However, it can be summed up that all models describe that students should undergo the four phases as suggested by Polya. It is important that mathematics educators first need to encourage, motivate and engage students with suitable mathematics problems that suit their abilities and skills.

The use of open-ended problems and encouraging students to explain the strategies and methods in solving mathematics problems is more pedagogically challenging among mathematics teachers (Johnson & Cupitt, 2004; McDonald, 2009). This method allows students to improve their abilities in problem solving and HOTS through engaging with appropriate mathematics problems. However, the use of open-ended problems and HOTS depend on the ability of the students in problem solving (Asami-Johansson, 2015). To encourage and motivate students, the problems should be at the levels that enable learners to solve at least some of the problems, to some extent (Bergqvist, 2011). In this study, the lecturer in the experimental class provided encouragement and motivation during problem solving activities through the use of fun and practical problems, teamwork and oral encouragement. Figure 1 shows the model of mathematical problem solving used for this study.

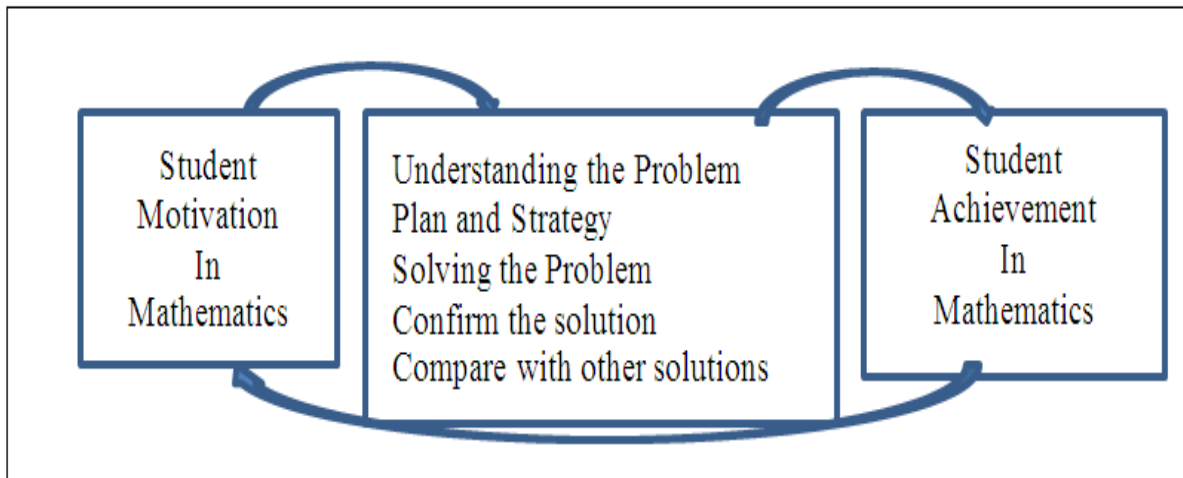


Figure 1. Conceptual Model of Mathematical Problem Solving in this Study

Higher Order Thinking Skills

During the last two decades, mathematics education experts have emphasised more on the learning of mathematics conceptually through problem solving and HOTS. One of the challenges is that there is little agreement concerning the operational definition for HOTS. Thomas and Thorne (2009) suggested that HOTS are above the level of memorising facts or giving back the memorised fact to someone, in the same way it was read or told. Bloom (1956) categorised thinking skills, beginning from the concrete and progressing to the abstract, in hierarchy from knowledge, comprehension, application, analysis, synthesis to creative. McBain (2011) considered the last three levels of Bloom’s Taxonomy — which are analyze, synthesis, and creative — as HOTS. However, the Malaysian Ministry of Education (2014) defined HOTS as the ability to apply knowledge, skills and values to make reasoning and reflection to solve problems, make decisions, innovate and strive to create something. Thus, based on this definition, the four levels which are applying, analyzing, evaluating and creating of the Revised Bloom’s Taxonomy are categorised as HOTS (Table 1).

Table 1: Categorisation of Higher Order Thinking Skills

Level	Explanation
Applying	Using the knowledge, skills and values in different situations
Analysing	Break down the information to better understand the relationship between the division
Evaluating	Make judgments and decisions using the knowledge, experience, skills and values and justify
Creating	Produce a product or idea or create and innovative methods

(Source: Malaysian Ministry of Education, 2014)

The following problem, which was an item of the Trends of International Mathematics and Science Study assessment (TIMSS Advanced, 2015), reflects the required skills:

Two mathematical models are proposed to predict the return y , in dollars, from the sale of x thousand units of an article (where $0 < x < 5$). Each of these models, P and Q, is based on different marketing methods.

Model P: $y = 6x - x^2$

Model Q: $y = 2x$

For what values of x does model Q predict a greater return than model P?

- A. $0 < x < 4$ B. $0 < x < 5$ C. $3 < x < 5$ D. $3 < x < 4$ E. $4 < x < 5$

In this problem, HOTS refers to students' ability to make the inequality $6x - x^2 < 2x$ and solving it by using a sign-chart to find $(-\infty, 0) \cup (4, +\infty)$ as the answer of this inequality. Finally, according to the limitation of the domain ($0 < x < 5$), they should be able to choose $4 < x < 5$ as the answer to this problem. Also, if students understand the topic of inequality conceptually, they can find the answer easily by testing the two values, which are $x = 4$ and $x = 5$ to find that $4 < x < 5$ is the answer. It is important that the categorisation of problems in different levels of the revised Bloom's Taxonomy are strongly related to the ability of the students in mathematics.

It is interesting to explore the differences in the performance between genders, which is creating a greater gap in Malaysia. Males and females are different psychologically. For example, female students are more shy compared to male students (Zakaria, Zain, Ahmad, & Erlina, 2012). Since childhood, male students are known to be more natural at recognising problems. However, their concern in solving the problem is lower than female students, who tend to provide more effort towards problem-solving (D'Zurilla, Maydeu-Olivares, & Kant, 1998). In Malaysia, the performance of female students in mathematical problem solving is better than male students. In all Trends of International Mathematics and Science Study (TIMSS) assessments from 1999 to 2015, eighth grade female students obtained higher marks than male students. Gilleece, Cosgrove and Sofroniou (2010) clarified that female students are better in problem solving and HOTS compared to male students. Some studies showed that male students are more successful in mathematics problem solving and HOTS (OECD, 2003), whereas other studies reported that there is no significant difference between genders (Areepattamannil & Kaur, 2013).

With the right content knowledge and PCK, educators can improve students' HOTS through the problem-solving method based on the abilities of students. The purpose of this study was to investigate the impact of a lesson study, as a professional development programme for mathematics' lecturers, on students' achievement in problem solving and HOTS.

Theoretical Framework

Effective teaching and learning is the interaction between the quality of three components, namely, educator's knowledge, teaching materials and students' abilities. The lesson study has been found to be an effective professional development programme to improve educators' knowledge through the sharing of their knowledge, skills and experiences by emphasising on problem solving and HOTS, based on the students' abilities (Demir, Czerniak, & Hart, 2013; Fujii, 2016; Mon et al., 2016). In fact, the lesson study helps educators to improve their PCK and utilise it in seeing mathematics "through the eyes of their students" (Fernandez, Cannon, & Chokshi, 2003, p.179). In this educational method, educators try to consider the problems on such levels that every student would be able to solve at least some of the problems (Bergqvist, 2011), in order to increase the ability of students in learning mathematics conceptually through practising and engaging with problem solving.

The materials used by mathematics educators in teaching are important in improving the ability of learners in problem solving and HOTS, as well as creating their interest to learn mathematics. In the lesson study, educators, through collaborative work and the sharing of their knowledge and experiences, produce suitable materials in the research lessons such as exercises, problems and practical problems through the linking of different topics (Khalid, 2017; Mon et al., 2016). The research lessons are the key component of the lesson study (Lewis, 2002; McDonald, 2009). Figure 2 illustrates the theoretical framework of this study.

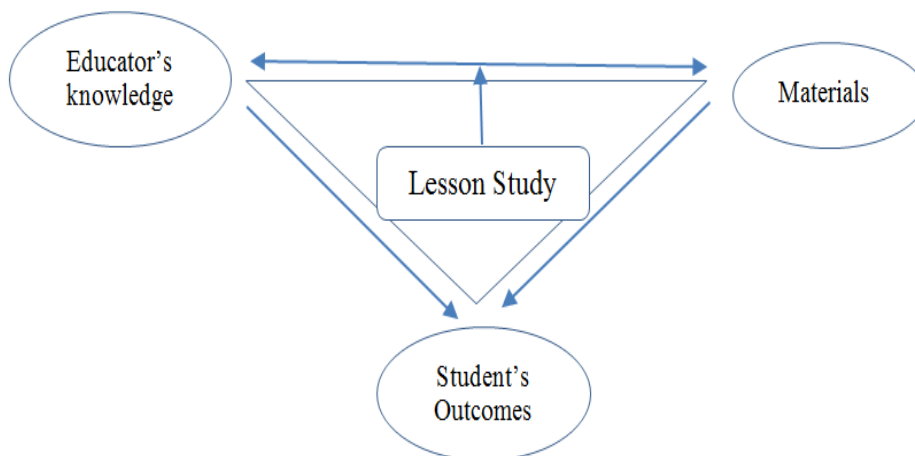


Figure 2. Theoretical Framework for the Study



Methodology

This quasi-experimental research with one control group was conducted during the first semester of the academic year of 2018–2019. The Foundation Centre students of a Malaysian public university were chosen as the subjects for the study. Several other universities also offer foundation programmes and the students are almost similar in their qualifying entrance grades. Students pursuing foundation or other pre-university education programmes are selected based on their high school performance. Thus, choosing one particular university for this study would also reflect, to some extent, the impact of such intervention to students of other universities.

Four male and five female lecturers taught mathematics in this centre. Initially, eight of the mathematics lecturers and two physics lecturers voluntarily agreed to participate in this study but later, a mathematics lecturer and a physics lecturer withdrew from participating due to their time constraints. Thus, the lesson study group for this study involved seven mathematics lecturers, a physics lecturer, and the researcher. The physics lecturer provided practical mathematics problems for the research lessons since she knew what mathematics needs to be applied in studying physics. Meanwhile, the researcher played several roles such as the coordinator, discussion leader, and lesson study group member. Once permission was obtained from the Director of the Foundation Centre and the Ethical Committee of the university, all lecturers and students who were participants of this study were asked to sign the consent letter. In this centre, 20 groups with 952 students (326 males and 626 females) studied the course, Mathematics 1, during the first semester of 2018/2019 and out of this population, two classes with a total of 95 students were selected at random as the experimental group with 50 students (16 males and 34 females) and the control group with 45 students (16 males and 29 females). Finally, the results of 44 students (16 males and 28 females) and 42 students (14 males and 28 females) were analysed for the lesson study and control groups respectively, because only this number of participants took all the tests given in the study.

The lesson study group members collaboratively planned, designed and discussed to prepare five research lessons about the topic on function. After that, a lecturer was randomly chosen using the fish-bowl technique to teach the research lessons to the experimental group. The experimental class was student-centred and students were engaged with problem solving activities individually and in a team during the class. Meanwhile, the lecturer monitored the students' learning, guided, assessed and confirmed the students' solutions. In this class, students discussed the different solutions for the problems given and learned new methods, skills and strategies to improve their problem solving ability and HOTS. In the control group, the same lecturer taught the same topics but conducted the class using a traditional approach. The class was more lecturer-centred and emphasised on routine exercises, or referred to as exercise solving. The instructional time for these topics was five weeks and students underwent five mathematics problem solving tests. Test one was conducted before the onset of the study,

while test two and test three were conducted during the five weeks of teaching. Test four was conducted after all lessons were completed and test five was conducted one month after the lesson study ended to determine the students' retention of what was learnt.

The instruments used in this study were tests developed by the researcher and five of the lecturers who were involved in the lesson study. Tests one, four and five were each comprised of 12 questions, while tests two and three were comprised of four questions. The tests were developed based on the TIMSS Advanced items, books and other resources. Although English is the language of instruction in the foundation centre and all textbooks are in English, the Bahasa Melayu (Malaysian Language) version was also included to ensure students did not have problems understanding the questions. Back to back translation was done by two experts from the English Language Department of the university, while four experts in the area of mathematics and mathematics education confirmed the preciseness of the translations. The validity of these tests were confirmed by seven experts in mathematics and mathematics education. These tests were also endorsed by some experts in the Research Management Centre (RMC) of the university. The reliability was ascertained using the Alpha Cronbach test with scores of more than 0.70 for all tests, which indicate an acceptable internal consistency.

The scoring for the questions is based on several criteria. If the student provided illogical and incorrect answers or did not respond, the score given was 0. A score of 1 was given if the student showed some understanding of the problem, as indicated in the phases of problem solving by Polya (1957). Likewise, if the student showed understanding and came up with a method for solving the problem although it may have errors, a score of 2 was given. A completely correct answer was given a score of 3. Thus, the minimum and maximum scores for tests with 12 questions are 0 and 36, and 0 and 12 for tests with four questions. Each student's answer sheet was scored by two evaluators. If there were differences between the scores given by the two evaluators, the final score for the student was calculated as follows. If the first and second lecturers gave scores a and b for a student, then the final score (s) for this student was, $s = \left[\frac{a+b+1}{2} \right]$, where $[]$ is the symbol of integral part. For instance, for scores of 18 and 20, the final mark was $\left[\frac{18+20+1}{2} \right] = [19.5] = 19$.

The problems in each test were categorised according to the three levels of HOTS in the revised Bloom's Taxonomy, namely applying, analysing and evaluating based on Table 1 and was confirmed by three experts in the university. Therefore, the researcher compared the results of the students in problem solving according to the different weight of scores for the three levels of Bloom's Taxonomy, between the lesson study and control groups. The purpose was to score the performance of students for each item according to the complexity of the items. The scores of all five problem solving tests were compared after this method of scoring (all scores ranged between 0 and 3). So, for tests one, four and five, the weight $\frac{3}{48}$ was considered for questions

1, 3, 7 and 10 (level of applying). Similarly, for questions 2, 4, 9 and 11 (level of analysing), the weight considered was $\frac{4}{48}$, and finally for questions 5, 6, 8 and 12 (level of evaluating), the weight $\frac{5}{48}$ was considered. Therefore, if the scores of a student for the 12 items were a_1, a_2, \dots, a_{12} respectively, the final score a was calculated as $a = [(a_1 + a_3 + a_7 + a_{10}) \times 3 + (a_2 + a_4 + a_9 + a_{11}) \times 4 + (a_5 + a_6 + a_8 + a_{12}) \times 5] \div 48$. For test two, the researcher used the weight $\frac{2}{12}$ for items 2 and 4, the weight $\frac{3}{12}$ for item 1, and the weight $\frac{5}{12}$ for item 3. Thus, if the scores of a student for items 1, 2, 3 and 4 were a_1, a_2, a_3 and a_4 respectively, the final score a is calculated according to the relation $a = (3a_1 + 2a_2 + 5a_3 + 2a_4) \div 12$. Also, in test three, the weight $\frac{2}{12}$ was considered for items 1 and 2, the weight $\frac{3}{12}$ for item 3, and the weight $\frac{5}{12}$ for item 4. So, if the scores of a student for items 1, 2, 3 and 4 were a_1, a_2, a_3 and a_4 respectively, his/her final mark was calculated as $a = (2a_1 + 2a_2 + 3a_3 + 5a_4) \div 12$. Therefore, according to these calculations, the minimum and maximum scores are 0 and 3 respectively for all five tests. As an example, Table 2 shows the level, coefficient and the item of Test 4.

Table 2: Level and Item of Test Four

Number	Level	Coefficient	Item
1	Applying	$\frac{3}{48}$	$f = \{(2,3), (2, a + 2b), (-1, 6), (4, 5), (4, 7 - 2a)\}$ is a function. Find the value of b .
11	Analysing	$\frac{4}{48}$	If $f(x) = 1 + \sqrt[3]{x - 2}$ then the function $f^{-1}(x)$ is? a. $f^{-1}(x) = x^3 + 3x^2 - 3x - 1$ b. $f^{-1}(x) = x^3 - 3x^2 + 3x + 1$ c. $f^{-1}(x) = 2x^3 - 3x^2 - 4x + 1$ d. $f^{-1}(x) = x^3 - 3x^2 + 3x - 3$
8	Evaluating	$\frac{5}{48}$	Let m be a non-zero constant. Find the two x -values where the graphs of $y = 10^{6m}x$ and $y = \frac{x^2}{10^{5m}}$ intersect.

The scores of all tests, namely test one, test two, test three, test four and test five, ranged between 0 and 3. The data was analysed by using the independent t-test, ANOVA and repeated measure ANOVA.

Findings

The normality of distribution of scores was checked using the Kolmogorov-Smirnov test, Shapiro-Wilk test, Skewness and Kurtosis. Table 3 shows the normality of scores for the different groups.

Table 3: Test of Normality of Scores

Test	Group	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistics	df	Sig	Statistics	df	Sig
Test 1	Lesson Study (LS)	0.14	44	0.03	0.97	44	0.26
	LS (Male)	0.19	16	0.14	0.89	16	0.05
	LS (Female)	0.14	28	0.19	0.96	28	0.34
	Control	0.11	42	0.20	0.98	42	0.77
	Control (Male)	0.12	14	0.20	0.96	14	0.68
	Control (Female)	0.12	28	0.20	0.97	28	0.57
	Total	0.06	86	0.20	0.99	86	0.97
Test 2	LS	0.11	44	0.20	0.94	44	0.02
	LS (Male)	0.11	16	0.20	0.96	16	0.70
	LS (Female)	0.12	28	0.20	0.93	28	0.07
	Control	0.11	42	0.20	0.96	42	0.11
	Control (Male)	0.17	14	0.20	0.91	14	0.14
	Control (Female)	0.12	28	0.20	0.93	28	0.07
	Total	0.11	86	0.20	0.97	86	0.06
Test 3	LS	0.27	44	0.00	0.80	44	0.00
	LS (Male)	0.35	16	0.00	0.70	16	0.00
	LS (Female)	0.27	28	0.00	0.84	28	0.00
	Control	0.17	42	0.00	0.93	42	0.02
	Control (Male)	0.13	14	0.20	0.95	14	0.62
	Control (Female)	0.26	28	0.00	0.91	28	0.02
	Total	0.16	86	0.00	0.93	86	0.00
Test 4	LS	0.08	44	0.20	0.97	44	0.27
	LS (Male)	0.22	16	0.04	0.85	16	0.01
	LS (Female)	0.13	28	0.20	0.95	28	0.20
	Control	0.06	42	0.20	0.99	42	0.88
	Control (Male)	0.16	14	0.20	0.95	14	0.60
	Control (Female)	0.10	28	0.20	0.98	28	0.89
	Total	0.06	86	0.20	0.99	86	0.52
Test 5	LS	0.12	44	0.10	0.96	44	0.09
	LS (Male)	0.12	16	0.20	0.97	16	0.86
	LS (Female)	0.16	28	0.07	0.91	28	0.03
	Control	0.11	42	0.20	0.98	42	0.48
	Control (Male)	0.12	14	0.20	0.97	14	0.92
	Control (Female)	0.14	28	0.17	0.97	28	0.66
	Total	0.09	86	0.16	0.98	86	0.14

The Kolmogorov-Smirnov Test showed that all scores for the different groups were normally distributed because the significant-value is greater than 0.05. However, for the significant-value of less than 0.05 for some groups, the normality of these groups were further checked as shown in Table 4.

Table 4: Test of Normality by Kurtosis and Skewness

Test	Group	Number	Skewness (Error)	Z-score Skewness	Kurtosis (Error)	Z-score Kurtosis
Test 1	LS	44	-0.04 (0.36)	-0.11	-0.79 (0.70)	-1.13
Test 3	LS	44	-0.49 (0.36)	-1.37	1.13 (0.70)	1.61
	LS (Male)	16	-1.02 (0.56)	-1.81	2.00 (1.09)	1.83
	LS (Female)	28	-1.07 (0.64)	-1.67	0.59 (0.86)	0.69
	Control	42	-0.44 (0.37)	-1.22	0.93 (0.72)	1.30
	Control (Female)	28	0.20 (0.44)	0.46	-0.32 (0.86)	-0.37
	Total	86	-0.39 (0.26)	-1.50	-0.68 (0.51)	-1.33
Test 4	LS (Male)	16	-1.57 (0.87)	-1.82	1.18 (1.09)	1.08

Kim (2013) and Mishra et al. (2019) explained that by using skewness and kurtosis, a Z-score could be obtained by dividing the skew values or excess kurtosis by their standard errors as follows:

$$Z = \frac{\text{Skew value}}{\text{Standard error}} \text{ or } Z = \frac{\text{Excess kurtosis}}{\text{Standard error}}$$

Then we can discuss the normality of data according to the size of the sample as follows:

- If the sample size is less or equal 50 ($n \leq 50$) and $|Z - score| < 1.96$, then data is normally distributed.
- If the sample size is between 50 and 300 ($50 < n \leq 300$) and $|Z - score| < 3.29$, then data is normally distributed.
- If the sample size is more than 300 ($n > 300$) and the values of skewness and kurtosis without considering the Z-scores are between -2 and 2, then data is normally distributed.

Since the absolute values of the Z-scores for all groups in Table 4 are less than 1.96, thus, the scores do differ from normal distribution. Table 5 shows the descriptive statistics of all groups.

Table 5: The Descriptive of Scores for all Groups

Test	Group	Number	Mean	Standard Deviation	Minimum	Maximum
Test 1	LS	44	1.41	0.34	0.75	2.16
	LS (Male)	16	1.39	0.31	0.81	1.77
	LS (Female)	28	1.42	0.35	0.75	2.16
	Control	42	1.57	0.45	0.52	2.56
	Control (Male)	14	1.58	0.64	0.52	2.56
	Control (Female)	28	1.56	0.32	0.93	2.20
	Total	86	1.48	0.40	0.52	2.56
Test 2	LS	44	1.88	0.47	1.17	3.00
	LS (Male)	16	1.79	0.35	1.25	2.50
	LS (Female)	28	1.93	0.53	1.17	3.00
	Control	42	1.60	0.48	0.83	2.67
	Control (Male)	14	1.70	0.49	1.17	2.67
	Control (Female)	28	1.55	0.47	0.83	2.33
	Total	86	1.74	0.49	0.83	3.00
Test 3	LS	44	2.17	0.25	1.33	2.58
	LS (Male)	16	2.14	0.28	1.33	2.33
	LS (Female)	28	2.18	0.24	1.58	2.58
	Control	42	1.55	0.29	0.67	2.00
	Control (Male)	14	1.47	0.34	0.67	2.00
	Control (Female)	28	1.87	0.25	1.08	2.00
	Total	86	1.86	0.41	0.67	2.58
Test 4	LS	44	1.90	0.43	0.75	2.62
	LS (Male)	16	1.92	0.39	0.75	2.56
	LS (Female)	28	1.89	0.45	1.04	2.62
	Control	42	1.48	0.32	0.83	2.16
	Control (Male)	14	1.44	0.37	0.83	2.06
	Control (Female)	28	1.51	0.43	0.87	2.16
	Total	86	1.70	0.43	0.75	2.62
Test 5	LS	44	1.94	0.32	1.29	2.44
	LS (Male)	16	1.85	0.27	1.29	2.31
	LS (Female)	28	1.99	0.34	1.35	2.44
	Control	42	1.76	0.33	0.92	2.46
	Control (Male)	14	1.51	0.31	1.00	2.08
	Control (Female)	28	1.61	0.34	0.92	2.46
	Total	86	1.76	0.37	0.92	2.46

The result of the independent t-test in Table 6 shows that there is no significant difference between the mean scores of the lesson study and control groups in test one ($t(84) = -1.92$, $p = 0.06 > 0.05$).

Table 6: T-test for Test One

Group	Number	Mean	Standard Deviation	t	df	Sig
Lesson Study	44	1.40	0.33	-1.92	84	0.06
Control	42	1.56	0.44			

The repeated measures one-way ANOVA was conducted to compare the performance of students in problem solving and HOTS between the lesson study and control groups during the period of this study. Table 7 shows the homogeneity of variances across the lesson study and control groups because all the p-values are greater than 0.05.

Table 7: Levene's Test for Equality of Variances

Test	F	df1	df2	Sig
Test1	0.99	1	84	0.32
Test2	0.04	1	84	0.85
Test3	0.79	1	84	0.38
Test4	1.92	1	84	0.17
Test5	0.37	1	84	0.55

For this analysis the null and alternative hypothesis were as follows:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Not all means are equal

The results of the independent t-test in Table 6 show there were no significant differences between the means of the lesson study and control groups in test one, thus, there was no need to consider test one as a covariate in this analysis. Table 8 shows the Mauchly's Test of Sphericity for homogeneity of covariance among the dependent variables.

Table 8: Mauchly's Test of Sphericity

Mauchly's W	Approx. Chi-Square	df	Sig	Epsilon		
				Greenhouse-Geisser	Huynh-Feldt	Lower-bound
0.53	51.81	9	0.00	0.82	0.87	0.25

Since the value of $p = 0.00$ is less than 0.05, thus the use of other replacement tests such as Greenhouse-Geisser or Huynh-Feldt were applied. The Huynh-Feldt was used because the ϵ -value is greater than 0.75. Table 9 shows the results of this test.

Table 9: Test of Within-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Time	6.47	3.46	1.87	15.31	0.00	0.15
Time*Group	7.25	3.46	2.10	17.16	0.00	0.17
Error (Time)	35.48	290.61	0.12			

In Table 9, the p-value ($p = 0.00$) is less than 0.05. Therefore, the alternative hypothesis was accepted. It means the performance between the lesson study and the control group were not the same. The Bonferroni Post Hoc was conducted to find whether there were significant differences between the two methods at each time point. Table 10 compared the effectiveness of the two educational methods in the different periods of time.

Table 10: Pairwise Comparisons

Test (I)	Test (J)	Mean Difference (I-J)	Std. Error	Sig.
1	2	-0.25	0.06	0.00
	3	-0.37	0.05	0.00
	4	-0.21	0.05	0.00
	5	-0.27	0.05	0.00
2	3	-0.12	0.07	0.39
	4	0.05	0.06	1.00
	5	-0.02	0.06	1.00
3	4	0.16	0.05	0.01
	5	0.10	0.04	0.24
4	5	-0.07	0.03	0.27

Table 10 provides the significance level for differences at individual points based on the time the tests were given. A significant difference in problem solving and HOTS was established between the lesson study and control groups during test one and test two ($p = 0.00$), test one and test three ($p = 0.00$), test one and test four ($p = 0.00$), test one and test five ($p = 0.00$), and test three and test four ($p = 0.01$). It can be seen from the mean difference column that the students' ability in problem solving and HOTS in the lesson study group was significantly increased at these time points. Figure 3 shows the comparison of the mean scores between the lesson study and control groups in tests one to five, which indicates the lesson study approach is more effective at improving students' abilities in problem solving and HOTS rather than the conventional method.

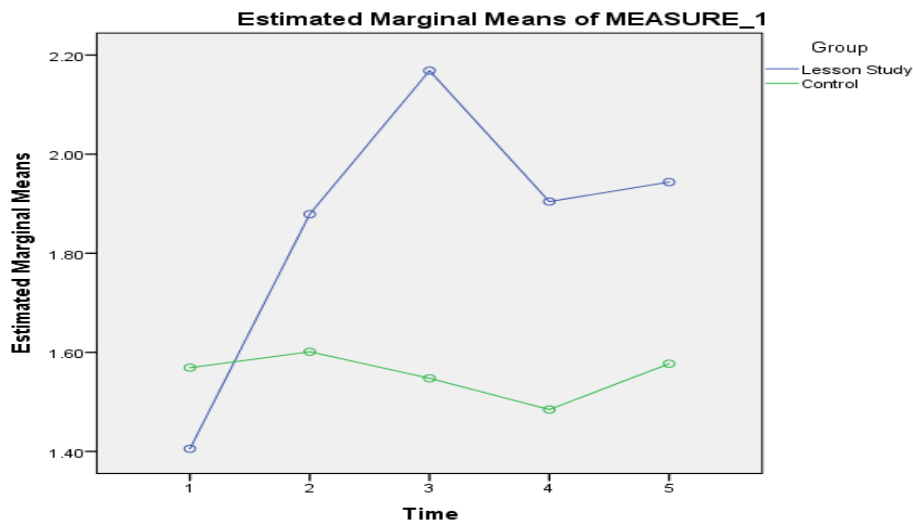


Figure 3. Comparison of Performance of Students in Problem Solving and HOTS

The analysis of repeated measures ANOVA was conducted to compare the effectiveness of the lesson study and conventional methods during the different periods of this study by gender. For this analysis, the null and alternative hypothesis were as follows:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Not all means are equal

Table 11 shows the homogeneity of variances across the groups by gender because all the p-values were greater than 0.05.

Table 11: The Homogeneity of Variances by Gender

Test	Levene Statistic	df1	df2	Sig
Test1	4.25	3	82	0.08
Test2	1.18	3	82	0.32
Test3	0.92	3	82	0.44
Test4	2.07	3	82	0.11
Test5	0.66	3	82	0.58

The results of the ANOVA test, as shown in Table 12, indicate that there were no significant differences between the means of the lesson study and control groups by gender in test one ($F(3, 82) = 1.23, p = 0.31, p > 0.05$). Thus, test one was not considered as a covariate in this analysis.

Table 12: The Results of ANOVA Test by Gender in Test One

Group	Number	Mean	SD	F	Sig
LS (Male)	16	1.39	0.31	1.23	0.31
LS (Female)	28	1.42	0.35		
Control Male	14	1.58	0.64		
Control Female	28	1.57	0.32		

Table 13 shows the Mauchly's Test of Sphericity for homogeneity of covariance among the dependent variables.

Table 13: Mauchly's Test of Sphericity

Mauchly's W	Approx. Chi-Square	df	Sig	Epsilon		
				Greenhouse-Geisser	Huynh-Feldt	Lower-bound
0.52	52.15	9	0.00	0.82	0.89	0.25

Since the value of $p=0.00$ is less than 0.05, we need to use other replacement tests such as the Greenhouse-Geisser or Huynh-Feldt. The Huynh-Feldt was applied because the ϵ -value is greater than 0.75 (Table 14).

Table 14: Test of Within-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig	Partial Eta Squared
Time	5.50	3.55	1.55	12.96	0.00	0.14
Time*Group	7.92	10.65	0.74	6.22	0.00	0.19
Error (Time)	34.80	291.08	0.12			

In Table 14, the p-value ($p = 0.00$) is less than 0.05. Therefore, the alternative hypothesis was accepted. It means the performance in the lesson study and conventional groups were not the same during the periods of this study. The Bonferroni Post Hoc was conducted to find whether there were significant differences between the two educational methods at each time point. Table 15 compared the effectiveness of the two educational methods at the different points of time.

Table 15: Pairwise Comparisons

Test (I)	Test (J)	Mean Difference (I-J)	Std. Error	Sig
1	2	-0.26	0.06	0.00
	3	-0.36	0.05	0.00
	4	-0.21	0.05	0.00
	5	-0.26	0.05	0.00
2	3	-0.10	0.06	0.85
	4	0.05	0.06	1.00
	5	0.00	0.06	1.00
3	4	0.16	0.05	0.04
	5	0.10	0.05	0.25
4	5	-0.05	0.03	0.93

Table 15 shows there are significant differences between the individual time points (based on the test time) by gender. There was a significant difference in problem solving and HOTS between the lesson study and control groups by gender during test one and test two ($p = 0.00$), test one and test three ($p = 0.00$), test one and test four ($p = 0.00$), test one and test five ($p = 0.00$), and test three and test four ($p = 0.04$). The mean difference column shows that the students' ability in problem solving and HOTS was significantly increased at these time points. Figure 4 shows the comparison of the mean scores between the lesson study and control groups in the tests by gender. This diagram shows the lesson study approach is more effective at improving students' abilities in problem solving and HOTS than the conventional method and there is no significant mean difference between genders.

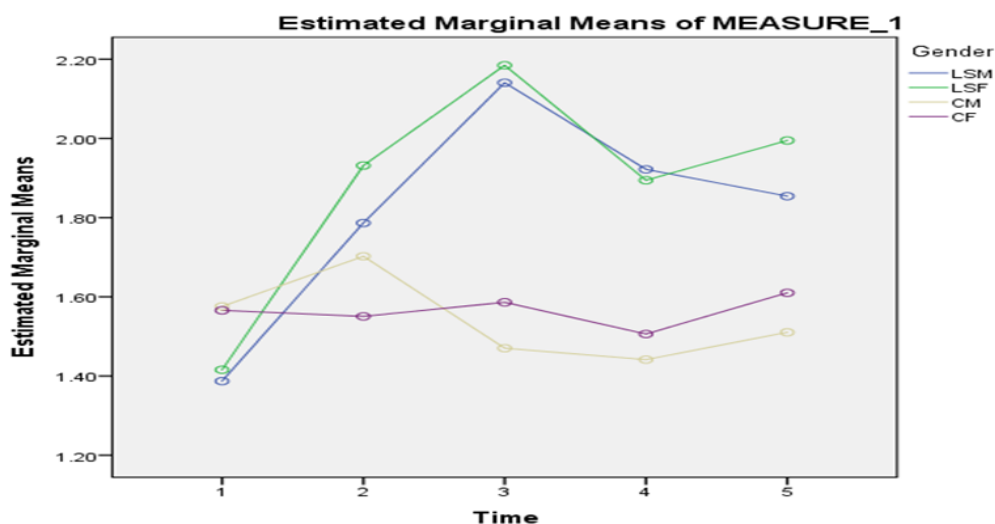


Figure 4. Comparison in Tests between Genders



Note: LSM=Lesson Study (Male), LSF=Lesson Study (Female), CM=Control (Male), CF=Control (Female)

Discussion and Conclusion

Many researchers (Intaros, Inprasitha, & Srisawadi, 2013; McDonald, 2009; Mon et al., 2016; Tambychik & Meerah, 2010) have explained that students normally receive mathematics materials which emphasise on the solving of routine exercises. Due to this, students have the impression that they only need to memorise the formulas, theorems, shortcuts and methods to apply in exercise solving and in preparing for examinations. In fact, they are seldom engaged with problem solving during their mathematics classes. Mathematics education experts faced questions such as “how can students learn mathematics meaningfully without doing the problem solving?”, “how can students experience the beauty of mathematics problem without problem solving?” and “how can students improve their HOTS without engaging with appropriate problems?”. During the last two decades, policy makers, administrators and mathematics education experts in Malaysia have tried to improve the quality of teaching among educators by emphasising on problem solving and HOTS (Alhassora, Abu, & Abdullah, 2017). One of the main challenges among mathematics educators is the design of good problem solving tasks that are original, non-routine and new to the students (Doorman et al., 2007).

The results of this study showed that a lesson study is an effective educational method in teaching mathematics because prior to the implementation, there were no significant mean differences between the students' scores in the lesson study and control groups but during all time points, namely in tests two, three, four and five, there were significant mean differences between the lesson study and control groups. This indicates that students in the lesson study group showed better performance in problem solving and HOTS compared to the control group. The students in the lesson study group had the opportunity to engage with problems at different levels of the revised Bloom's Taxonomy, individually and in groups. Engaging learners with suitable and practical problems improved their interest and ability in HOTS. The lesson study, through collaborative work among lecturers and with emphasis on problem solving, encouraged students to practice and engage with problems. The lecturers themselves acquire rich learning opportunities when they successfully form a learning community with shared norms and values for supporting student learning (Horn, Garner, Kane, & Brasel, 2017; Horn & Little, 2010).

The students in the control group were taught using the traditional teaching method. They were not given opportunities to engage with problem solving, resulting in them feeling that mathematics is difficult and boring. They are often required to memorise lots of formulas, theorems and methods. For example, in the lesson study method, students easily learned many trigonometric formulas such as $\sin(\pi + \theta) = -\sin \theta$, $\cos(\pi - \theta) = -\cos \theta$ and $\tan(2\pi -$

$\theta) = -\tan \theta$ by using the trigonometric circle. However, in traditional method students, they had difficulty in memorising the trigonometric formulas and in applying them in problem solving. The study does highlight that a conceptual understanding about the concept and theory underlying the mathematics concepts not only improved students' power in problem solving and HOTS, but also increased their interest and self-confidence in learning mathematics.

One of the important benefits of the lesson study approach is the discussion about the different approaches in solving a given problem. When students discussed the different approaches, they learnt many new concepts, techniques and ideas that helped improve their power in problem solving. For example, in this study, students solved the problem "Find the value of $A = \sin 15^\circ + \cos 15^\circ$ " through different ways, such as the following:

$$i) \quad A = \sin 15^\circ + \cos 15^\circ = \sin(45 - 30) + \cos(45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30$$

$$+ \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \sqrt{\frac{3}{2}}$$

$$ii) \quad A = \sin 15^\circ + \cos 15^\circ \rightarrow A^2 = (\sin 15 + \cos 15)^2 = \sin^2 15 + \cos^2 15 + 2 \sin 15 \cos 15$$

$$\rightarrow A^2 = 1 + \sin 30 = 1 + \frac{1}{2} = \frac{3}{2} \rightarrow A = \sqrt{\frac{3}{2}}$$

iii) By using the formulas $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ and $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ students discussed that

$$\sin^2 15 = \frac{1 - \cos 2 \times 15}{2} = \frac{1 - \cos 30}{2} = \frac{1}{2} \left(1 - \frac{\sqrt{3}}{2} \right) = \frac{2 - \sqrt{3}}{4} \rightarrow \sin 15 = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\cos^2 15 = \frac{1 + \cos 2 \times 15}{2} = \frac{1 + \cos 30}{2} = \frac{1}{2} \left(1 + \frac{\sqrt{3}}{2} \right) = \frac{2 + \sqrt{3}}{4} \rightarrow \cos 15 = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$A = \sin 15 + \cos 15 = \frac{\sqrt{2 - \sqrt{3}}}{2} + \frac{\sqrt{2 + \sqrt{3}}}{2}$$

To find the value of A , students squared both sides of the above equation as follows:

$$A^2 = \frac{1}{4} (2 - \sqrt{3} + 2 + \sqrt{3} + 2\sqrt{2 - \sqrt{3}}\sqrt{2 + \sqrt{3}}) = \frac{3}{2} \rightarrow A = \sqrt{\frac{3}{2}}$$



Encountering different approaches to solve a given problem allows students to learn conceptually. By trying different methods, students were able to connect many related concepts such as trigonometric identities, formulas related to the sum and difference of angles, double angles and half angles. In this way, students get to improve their abilities in problem solving and HOTS by linking to different concepts. As posited by Khalid (2017), and Mon et al. (2016), the materials that were used in the mathematics classes taught using the traditional method cannot improve the skills of students in problem solving and HOTS. The research lessons are the key component of the lesson study process (Lewis, 2002).

In the lesson study approach and through collaborative work the lecturers planned the research lessons to minimise the need to memorise and to allow students to learn conceptually and meaningfully. For example, the lecturer's definition of even and odd functions in the traditional group did not refer to the domain of the functions, so students ended up superficially memorising the two properties $f(-x) = f(x)$ and $f(-x) = -f(x)$ to identify whether a function is even or odd. However, in the lesson study group, the lecturer discussed the domain of the even and odd functions and improved the definitions of even and odd functions.

For the problem "Determine whether the function $f(x) = \sqrt{x} + \sqrt{-x}$ is even or odd" in test three, students in the traditional group argued that this is an even function because this function satisfies in the condition $f(-x) = f(x)$ as $f(-x) = \sqrt{-x} + \sqrt{x} = \sqrt{x} + \sqrt{-x} = f(x)$ without understanding the properties of even and odd functions. However, a majority of students in the lesson study group would first identify the domain and the rule of the function f as $D_f = \{0\}$ and $f = \{(0, 0)\}$, so this zero-function is both even and odd. Therefore, when students understand the concept of even and odd functions, they can apply this knowledge to other mathematics topics such as the range of the functions, limit and integration. For instance, the properties of odd and even functions can be applied to the topic on integration to find the value of $\int_{-1}^1 \frac{x^5}{x^4 + \cos x} dx$ is zero because the function $f(x) = \frac{x^5}{x^4 + \cos x}$ is an odd function and the graph of this function is symmetric with respect to the point $(0, 0)$. The results of this study also showed that the lesson study improved students' mathematical problem solving and HOTS during the different time points. Regarding gender, no significant difference was found between male and female students.

The lesson study is an effective method to be applied at the pre-university and not just at the school level, as has been the focus of earlier studies. The lecturers at the foundation centre would be able to find significant opportunities to implement a lesson study where they can share their knowledge, skills and experiences to improve the students' abilities in mathematics problem solving and HOTS.



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