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Epistemological and Didactical Obstacle Exploration and Hypothetical Learning Trajectory in Learning of Three Dimensional Geometry.

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Geometry is one of the most important materials in learning mathematics. However, in reality not a few students who are taught the geometry concept have difficulty in understanding. One of the most difficult of geometric materials is the three dimensional, given to grade X high school students, especially concerning the distance concept. The research aims to find out the students' learning obstacle in studying three dimensional geometry, especially the distance concept, in terms of epistemological obstacle and didactical obstacle. Furthermore, the results of the study, will be used as a reference in preparing the HLT. Therefore it is expected to minimise learning obstacles for students studying geometry. This research used a descriptive qualitative method to find out student learning obstacles. The research was conducted in one high school of Tasikmalaya involving 32 students of class X.

Key words: *Learning, Geometry, Mathematics*

Introduction

Geometry is one of the most important material in learning mathematics. Geometric ideas are used to present and solve problems in other mathematical material and real-world situations (Midgett and Eddins, 2001). Basically students already know the geometry in everyday life such as lines, shapes and spaces. Geometry in mathematical learning looks easy; students can draw and calculate what is asked. However, in reality not a few students who are taught the geometry concept have difficulty in understanding. Some expert research concludes that high

school students also have little knowledge or experience about the properties of geometric spaces (Jiang, 2008). The difficulties of three dimensional geometry material are not only experienced by students but also teachers in teaching it.

Learning geometry in grade X of high school is placed most recently in the school year since the 1994 Curriculum supplementary 1999, the 2004 Curriculum (Competency Based Curriculum) and the 2006 Curriculum (Curriculum Level of Education Unit). In the 2013 curriculum, the three dimensional geometry material in Chapter 9 is an enrichment material. Looking at this paradigm, it is as if geometry received less serious attention. Geometry is also not regarded as something important because the presentation is only a small portion and the representation concepts are more oriented to the easy concept.

Here is an outline of the geometry material students learn ranging from elementary to high school, as well as clear and important implications from geometry with other materials in mathematics curriculums in Indonesia, (a) Measurement (Primary school): Measurement and geometry very much in line especially in determining formula of the circumference and area of shape, and also in calculating the volume of space; (b) Fraction (Primary school): Fraction material has relevance to geometry, because in primary school students learn fractions through parts of the form; (c) Integers (Primary school): The coordinate plane provides an interest because both positive and negative numbers are used in the translation of positions in planes and spaces; (d) Triangle and Quadrilateral (Grade VII); (e) Circle (Grade VIII); (f) Flat surface three dimension (Grade VIII): This material deals with the volume and surface area of cubes, blocks, pyramids, and prisms and their applications in everyday life [3], and (h) Three dimensions (Grade X): This material deals with the position of point, line, and plane, the distance between point, line, and plane, the angle by two lines, or line and plane, or two planes. In addition, this material also deals with the concept application of distance and angle in more analytical frameworks.

Based on the description, the geometry concept in curriculum cannot be underestimated and should get more serious attention. So much material is related to geometry as well as the implications on other materials. In fact there are still so many geometric implications on other materials at higher levels, including linear equations systems, quadratic functions, differential and integral, vector analysis, linear algebra, and many others.

By looking at the implications of the geometry concept on other materials in mathematics or outside mathematics, it is very important that students learn geometry . According to Van De Walle, (2008) geometry needs to be learned for the following reasons: (1) Geometry helps students to have complete beliefs about their world; (2) Exploration in geometry can help to develop problem-solving skills; (3) Geometry plays a major role in other fields. (4) Geometry is used by many people in everyday life. (5) Geometry is full of puzzles and fun.

If we look at the geometry material that has been presented by the teachers, this always refers to textbooks, where the material is adapted to the current curriculum. Unfortunately, however, the school's geometry curriculum to date includes very few of the true experiences. The primary and secondary school geometry curricula incorporates too many low-level experiences in which learners are only asked to learn the shapes' names. Then in high school, learners are expected to learn geometric reasoning when dealing with evidence. The typical elementary school curriculum keeps children at a low level of development, and then the high school curriculum makes no sense, expecting students to jump to higher levels of development. For most students this leap is impossible, and the development of their geometric thinking is impeded (Wahyudin, 2013). Therefore, it is very necessary to arrange teaching materials that are based on research on learning obstacles (barrier on learning) experienced by students in mathematics learning materials. Especially where the three dimensions in the grade X high school are concerned, students are sometimes hampered in solving problems encountered.

Learning obstacle is a barrier or difficulties that occur in the learning process. Student difficulties when learning are not always the same, this is because students experience different obstacles. In other words, these obstacles or difficulties can't be avoided as they are part of every learning process. Brousseau (1997) suggests the three types of learning obstacles are ontogenic, epistemological, and didactical obstacles that can occur in the learning process (Brousseau, 1997). Ontogenic obstacle is a type of difficulty related to the readiness of the child in learning. Epistemological obstacle is a type of learning difficulty that is more due to the limited context used when the first concept was studied (Duroux in (Brousseau, 1997)). Didactical obstacle is a learning difficulty caused by the didactic design conditions used or by teachers didactic intervention (Suryadi, 2015). This research revealed obstacle learning, especially those that are epistemological and didactic on the distance of space.

In order to achieve a learning goal that has been set by the teacher, each student has their own unique learning trajectory. The different experiences in learning as well as the variety of mental objects formed from each experience have a great impact on the variation of student learning flows. Teachers should be able to facilitate the variety of learning flow that develop during learning. Teachers should also be able to predict the various possible student responses as a result of the didactic situation developed, so that the Hypothetical Learning Trajectory (HLT) can be the main reference facilitation process in the form of didactic and pedagogical interventions. An HLT is developed based on the objectives to be achieved, as well as the learning stages of a continuous range of didactic situations toward the achievement of goals (Clements and Sarama, 2004).

The importance of HLT can be analogous to the planning of travel routes, if we understand the routes on the way to the destination, then we can choose a good route to use. In addition, we can also solve the problems in travel if we understand the route. When designing learning activities, teachers should arrange a hypothesis action or student reaction at each learning stage. In the early stages of the lesson planning, the hypothesis is based on a pre-knowledge estimate that students already have and based on their experiences or lessons learned in the previous year.

In this study, researchers explored some student learning obstacles, especially in studying the concept of three dimensional geometry. The scope of the material discussed here includes the concept of the distance between the point to the plane. Researchers studied learning obstacles in terms of the epistemological obstacles and didactical obstacles. After that, the researchers hope this study will become the basis to develop HLT which focuses on the students' thinking activities in constructing a geometric concept so that the geometry learning is easily understood by the students.

Based on the previous description, the research aims to find out the students' learning obstacles in studying three dimensional concepts, especially the distance in three dimensional, in terms of epistemoligical obstacle and didactical obstacle. Furthermore, the results of the study will be used as a reference in preparing the HLT, so it is expected to minimise learning obstacles experienced by students studying geometry.

Experimental method

A descriptive qualitative method was applied in this research. Descriptive qualitative research aims to interpret the phenomena that occur in students (Moleong, 2012). Qualitative descriptive research is a study where the data collection is gathered via observation, interview, or questionnaire about the current situation and the subject under investigation (Ruseffendi, 2010). This case study research used a triangulation data collection technique which consisted of observation, interview, and also documentation. The respondents were 32 students of grade X in one of the State Senior High Schools in Tasikmalaya.

Results and discussion

With regard to the learning obstacle construct of epistemological obstacle and didactical obstacle, Suryadi states that epistemological obstacles are obstacles that arise as a result of a person's knowledge being limited to a particular context (Suryadi et al., 2010). If the person is

dealing with different contexts, the knowledge becomes unusable or he or she will have difficulty using it.

It was found that some students' difficulties were in the concept of space distance. The most common difficulties encountered are the students' difficulties in determining the projection of points to lines or planes. As an illustration, the researcher presented the following cases.

Case 1: Look at Figure 1. Point P lies on segment HB and FP ⊥ HB. Is point P right in the middle HB? Give a reason!
Case 2: Look at Figure 2, it is known that point P is the midpoint of HF and point Q is the midpoint of PF. Is the projection of point E on the plane APF located on the segment AQ? Give a reason!
Case 3: Look at Figure 3, it is known that each points P, Q, and R are respectively the midpoints of the segments FG, HD, and AB. Is plane PQR an equilateral triangle? Give a reason!

Figure 1. Cube for case 1.

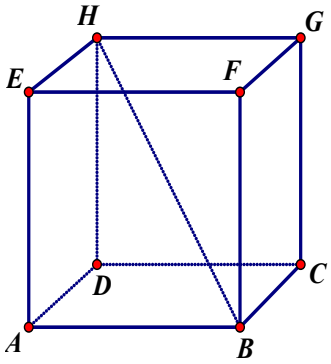


Figure 2. Cube for case 2.

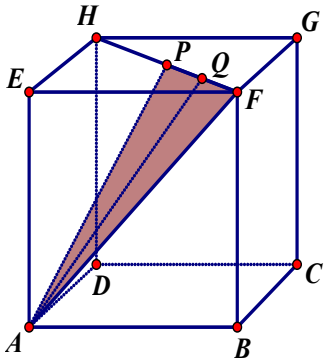
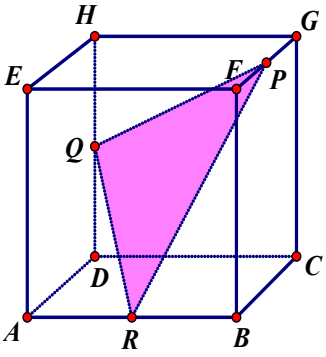


Figure 3. Cube for case 3.



Based on the answers of 32 students on case 1, it is indicated that all students answered "yes." Most students were fixated on the image, they were fooled by the image display. They assumed that if FP ⊥ HB then point P would be right in the middle of HB. Even some of them did not give a reason. Therefore, based on the student response for case I, the students still did not understand the point projection on the line, and this was a problem that needed to be solved.

Based on answers from 32 students on case 2, it showed that as many as eight students (25.0%) answered "no." The students who answered "no" did not provide the exact reason. They seemed to think only that the projection of point E on the plane APF lay on the segment AP. 24 students (75.0%) answered "yes." Most of them gave the answer "because the projection point E will be on the line AQ." They did not give clear reasons for their answers. But there were some students who gave the reason, "because the segment AQ cuts the plane

APF with the same area width." From several variations of the students' answers, it was concluded that the students also did not correctly master the concept of projection of a point on the plane, resulting in errors in geometrical concept. Thus, the aspects that lead to weak mastery of student geometry concepts need to be explored in more depth, especially regarding the concept of point projection to the line and the projection point to the plane. It was predicted more broadly that it was likely that students would have difficulty projecting a line on a plane because to project the line in the field involves more complex concepts of geometry.

Based on the answer of 32 students on case 3, the results show that 31 students (96.88%) answered "no." Only one student answered, "do not know." All of the students who answered "no" generally stated that the triangle PQR was a right triangle in Q. From the picture it is seen that the triangle PQR is a right triangle in Q, whereas when observed again, it turns out that the sides of the PQR triangle have the same length, this means that the triangle PQR is an equilateral triangle. From all the students' answers, the researcher concludes that the students' visualisation ability in looking at a whole space is not mastered very well. This greatly affects students when solving problems about the distance concept in the space, especially when the student transforms the shape which is described on three-dimension into two-dimension. Thus, the aspects that are causing the weakness of the student's visualisation in looking at the space need to be explored in more depth.

Based on the facts found, the researchers tried to analyse high school math books containing information about three dimensional material, especially about the distance concept on the space. The books analysed are generally used by teachers as textbooks or teacher handbooks in teaching three dimensional concepts. The books are used in accordance with the current curriculum, starting from the 1994 Curriculum 1999 Supplementary, the 2004 Curriculum (KBK), the 2006 Curriculum (KTSP) and the Curriculum 2013. The analysis aims to gain an overview of the given material flow to students as one of the studies on the didactical obstacles. By analysing the four books, the researcher hopes to find the answers of the problems related to students learning obstacles when studying the distance concept on the three dimensions. Each book respectively is among the books used as teacher handbooks in 2000 (Noormandiri and Sucipto, 2000), 2004 (KBK) (Rustiadin, and Agus, 2004), 2006 (KTSP) (Siswanto, 2006), and 2013 (Curriculum 2013) (Kementerian Pendidikan dan Kebudayaan, 2013).

In Noormandiri and Sucipto, (2000), the three dimensional material was given in the third grade of high school in the last chapter of the second quarter. The order of the three dimensional material in the book begins with the slicing of space, the perpendicular line, the projection of the line on the plane, and the last material was about the angle. When we look at the 1994 Supplementary Curriculum 1999, projected material is given after the distance

material. This indicated that when learning the concept of distance in the wake of space, the concept of projection is not used. This resulted in some examples of problems presented still providing examples of simple problems and tending not to apply the concept of projection in solving it.

Problems given about distance concepts can still be solved without using the concept of point projection or line projection either on the line or on the field in the wake of space. So naturally, if the projection material is given after the material distance on wake up space. However, on the other hand the researcher sees that the exercises about the concept of distance given to the students in this book are not as simple as the examples given. To solve the problem given as an exercise, students are required to master the concept of projection first. In fact, projection material is given after the distance material is given, so it is possible that many students may have difficulty in solving the problems.

In Rustiadin, and Agus, (2004), projection material is not discussed at all. The projection concept is not used in determining distance from point to line, or distance from point to plane, and others. In the material, the concept of projection is not introduced at all to the students, whereas the concept of point projection of the line is one of the difficulties students experience in finding the distance on the three dimensions.

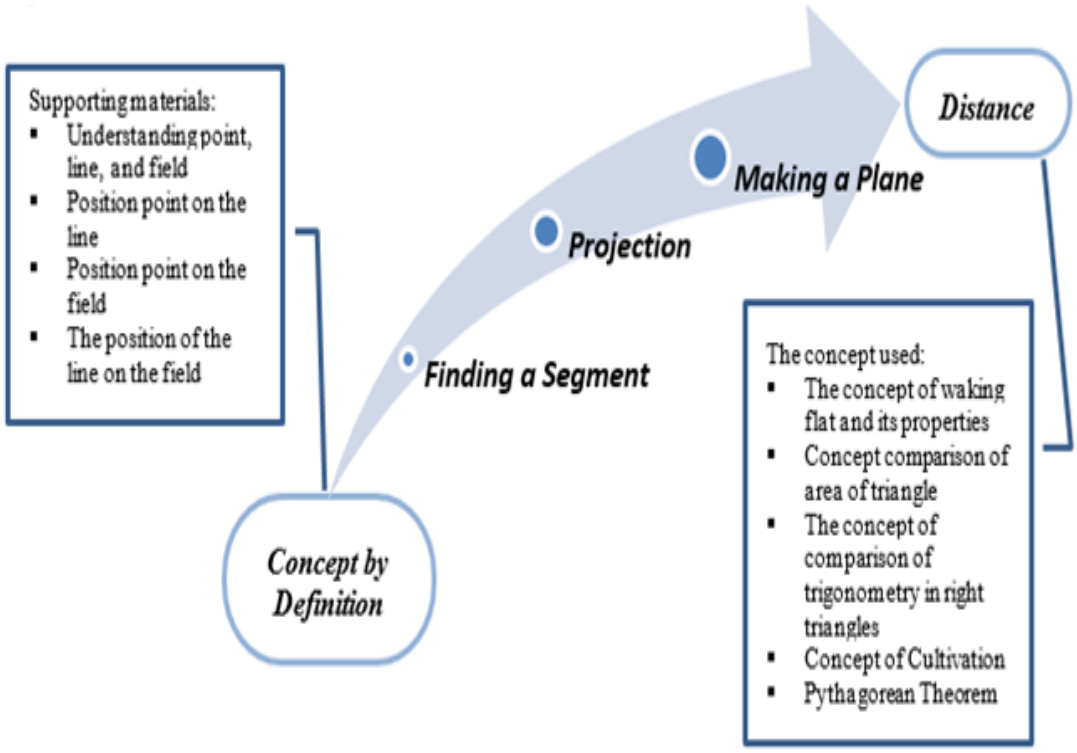
In Siswanto, (2006), before students discuss material about the distance in dimension three, students are first directed to discuss the concept of point projection to the field. However, the concept of projection described in book three is merely a theoretical explanation. Students are introduced to terms such as projection point, projector, and projection field. When students are only introduced to the theoretical projection concept alone, it is likely that students' understanding of the projection concept will be less than optimal.

In Kementerian Pendidikan dan Kebudayaan, (2013), the concept of point projection of lines is presented through everyday problems. Students are faced with the problem of how to measure the distance between the penalty spot on the soccer field against the goal line. The material in (Kementerian Pendidikan dan Kebudayaan, 2013) described above provides a more complete explanation of the concept of point projection on the line when compared to the material in (Noormandiri and Sucipto, 2000; Rustiadin, and Agus, 2004; Siswanto, 2006). However, in (Kementerian Pendidikan dan Kebudayaan, 2013), the student's action process in understanding the concept of projection is still less than optimal. Students are generally able to understand the concept of projection as presented in (Kementerian Pendidikan dan Kebudayaan, 2013), but when students are faced with problems directly involving the concept of point projection on the line in the wake of space, students still have difficulty. This may be due to a lack of student learning experience (process of action) in reviewing the concept of projection. In addition, the limitations of the students' imaginations in imagining

the projected point on the line are clear. To explore the answers of these problems, the researcher tries to interview one of the high school X class teachers in Tasikmalaya with the aim of exploring information about the learning barrier experienced by students in studying the three dimensional material, especially about the concept of distance in waking space. Based on the results of the interviews, the information obtained reveals the difficulty in finding teachers teaching distance concepts, i.e. students have not been optimal in determining the projection of the point either on the line or on the field.

Based on the researchers' observations of several textbooks (teacher manuals) used by teachers since the 2004 Curriculum (KBK), Curriculum 2006 (KTSP), and Curriculum 2013, in general these have similarities. The learning path of the concept of distance between points to fields in the previous textbook starts from the concept by definition with supporting material: (a) Understanding point, line, and field; (b) Position point on the line and on the field; and (c) Position of the line on the field, and then the material continues to determine distance. Based on the learning path of distance material in the three dimensions, it turns out there is a considerable concept leap, between the starting and finishing points . At the beginning of the learning the students are introduced to the concept of point spacing of the line or point spacing to the field theoretically (concept by definition) and after this stage is completed, the teacher immediately gives the problem of finding the distance from point to field presented directly on waking space. Of course, this learning path will make it difficult for students to understand the concept of distance. Therefore, based on the study of obstacles to students' learning on the matter of distance between points and fields, researchers tried to compile the Hypothetical Learning Trajectory (HLT) as presented in Figure 4.

Figure 4. HLT of distance between points and fields



Based on Figure 4, the researchers compiled the HLT distance point material learning into the field. In this material, any learning begins by introducing students with problems related to the concept of distance from a point to a field. Furthermore, using 3D Cabri software, students are directed to investigate and observe to find a segment of the field (finding a segment) containing a projection of a point on the field (projection). After that, students are directed to the next stage of creating a field containing the point and segment of the line found in the previous stage, and ending by calculating the distance from the point to the field by using various concepts accordingly.

Conclusions

It was concluded that the learning obstacles experienced by students in terms of epistemological and didactical obstacles for 32 students lies in (1) Determining the location of the projection of a point on the plane, and (2) Creating and recognising the shape of a plane containing the point and loading the segment of the field (which contains the result of the projection point).

The researcher's observation of several textbooks (teacher manuals) used by teachers in general has similarities in the learning path. The learning flow of the concept of point spacing to lines and dots to fields in some textbooks before starts from the concept by definition to directly solve the problem of distance. Based on the flow of learning, it turns out there is a

leap of concept far enough. The learning path in this format will make it difficult for students to understand the concept of distance. Therefore, Hypothetical Learning Trajectory (HLT) should be prepared to observe students' obstacle learning on distance material, so that students are able to understand the concept of distance in dimension three more optimally. HLT on learning distance material between point and line includes: concept by definition, projection, making a plane, and distance, while for HLT, learning material regarding the distance between point with field includes: concept by definition, finding a segment, projection, making a plane, and distance.

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