

Senior High School Students' Different Cognitive Styles and Their Thinking Processes in Solving Mathematical Problems with Scaffolding

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The aim of this study was to reveal the thinking process of two students of Public Senior High School 1 Raha, Indonesia. One student had an impulsive cognitive style and the other a reflective cognitive style, in solving mathematical problems through the provision of scaffolding based on Polya stage. This was an explorative study using a descriptive-qualitative approach. The main instrument of the study was the researchers, and the auxiliary instrument consisted of three respondents to a cognitive style test, problem-solving task, and interview. Data were analyzed through three stages, namely: data reduction, data presentation, and conclusion. The study found that: (1) at the problem-solving stage, the two subjects tended to think conceptually, but only the impulsive subject provided with scaffolding could really understand the problem well; (2) at the stage of planning problem-solving, the impulsive subject's thinking types tended to be semi-conceptual, and the reflective one tended to think conceptually, but only the former was provided with scaffolding; (3) at the stage of problem-solving plan implementation, the impulsive subject's thinking types tended to lead to semi-conceptual thinking, and solved the problem in a hurry that the solution tended to be wrong, but with scaffolding, he solved the problems correctly; whereas the reflective subject tended to think conceptually and solved problems correctly without scaffolding; (4) at the stage of re-examination of the results of problem solving, the impulsive subject tended to think

computationally, whereas the reflective one tended to think conceptually.

Key words: *Thinking process, cognitive style, scaffolding, and mathematical problem solving.*

Introduction

Problem solving is the most important part of mathematics teaching. The aim for teaching problem solving is to prepare students to be able to solve various forms of mathematical problems. In addition, it is useful for knowledge acquisition, formation of ways of thinking, and for given problem solving (Sudia, 2013; Lambertus, *et al.*, 2016). Problem solving is defined as an effort to find a way out of a difficulty to achieve a goal that is not immediately achievable (Polya, 1973). Problem solving is the actualization of a mental activity consisting of a variety of cognitive skills and actions intended to get the right solution (Foshay & Kirkley, 2003). Therefore, the ability in solving problems tends to vary. Similarly, Solso, Maclin and Maclin (2008) and Lambertus *et al.* (2014) also suggest that problem solving is a focused, directed thinking to find a solution or a way out of a specific problem. In other words, problem solving is an effort to find a way or ways out of a problem without an immediate solution.

Problem solving requires various stages. One of the most commonly stages referred to in mathematical problem solving is Polya (1973) stages: (a) understanding the problem; (b) planning a way out ; (c) acting the planned way out, (d) reviewing the result of the problem solving. Based on the Polya steps above, students are expected to have a well-structured thinking flow.

Solving a problem is still considered difficult by the majority of students. This phenomenon also occurs in State High School 1 Raha (here and after, SMAN 1 Raha) as informed by the mathematics teacher of Year 10 of SMAN 1 Raha in an interview on the December 7, 2015. “Students are still difficult to solve mathematical problems, especially those related to daily life problems” and problems often encountered were related to trigonometry (translated). Thus, the ability of students to solve mathematical problems was considered necessary to be studied.

Since solving problems involves high-level thinking processes, developing mathematical problem-solving skills is not easy. Thinking is a mental activity involving the brain. Thinking also means mental efforts to understand something experienced, or finding a solution to the problem being faced. Thinking activities begin when there are doubts or questions to be answered or with issues, or problems that require solution (Sobur, 2003). Suherman (2005)

defines thinking as “the process of generating new mental representations through information transformation involving complex interactions between mental attributes such as judgment, abstraction, reasoning, imagination, and problem solving”. According to Suherman (ibid) there are three main components in thinking, namely: (a) thinking is a cognitive activity that occurs in the mind of a person, is not visible, but can be inferred based on apparent behaviours; (b) thinking is a process involving some manipulation of knowledge within the cognitive system; (c) thinking activity is aimed at producing a solution.

According to Hudoyo (1988), in the process of learning mathematics occurs the process of thinking, because a person is said to think if he is doing mental activities. There are three kinds of soft thinking processes: conceptual thinking, semi-conceptual thinking, and the computational thinking (Milda and Mubarokah, 2013). The conceptual thinking process is a thinking process that tends to solve the problem by using the concept that has been owned. Semi-conceptual thinking is a thinking process that tends to solve problems by using incompletely understood concepts, and thus, the solution is mixed with an intuitive solution. On the other hand, computational thinking is a process of thinking that generally solves a problem without using the concept but rather intuition. Milda and Mubarokah (2013) mentioned three indicators of the thinking process above as follows: (1) conceptual thinking process: able to express using own sentence what is already known about the matter, able to answer using own sentence what is asked by a question, tend to use concepts studied to then answer questions, and able to mention elements of the concepts completed; (2) semi-conceptual thinking process: less able to express in own sentence what is known about the matter, less able to express with the sentence itself asked in the question, in answering questions, tend to use incomplete concept that has been studied, not fully able to explain steps taken; (3) the computational thinking process: unable to express in own sentence what is known about the matter, unable to express in own sentence what is asked by question, tend to answer questions away from the learned concept, unable to explain steps taken.

It is very important that teachers attend to the thinking process of students in solving mathematical problems. If their thinking is known, the teacher can uncover how the process develops in their mind when they solve problems. There are a lot of facts in mathematical problem solving, from basic to higher education levels.

When students have difficulty in solving mathematical problems, they should be assisted or guided. To develop students' thinking process in solving mathematical problems, there are two important concepts in Vigotsky's theory (cited in Lambas, 2004), namely Zone of Proximal Development (ZPD) and scaffolding. Zone of Proximal Development (ZPD) is defined as the space between the level of actual development and the level of potential development. The actual level of development is the child's ability level to solve certain problems independently, while the level of potential development is the level of development

a child can achieved if mentored or assisted by a more mature or more competent person (Lambas, 2004).

Anghileri (2006) suggests three levels in the provision of scaffolding, namely: (1) environmental provisions; (2) explaining, reviewing and restructuring; and (3) developing conceptual thinking. The scaffolding in this study refers to the second and third levels of scaffolding proposed by Anghileri (*ibid*). The first level, environmental provisions, relates to the arrangement of the learning environment, free play and cooperation with peers (suitable for elementary school children). At the explaining, reviewing and restructuring level, students are asked to understand the problem, reflect, correct answers and rearrange the draft answers. At the third level, of developing conceptual thinking, students are asked to find other alternatives to solve problems and discussions about the answers they have made.

In general, every student has their own cognitive style. Cognitive style refers to the tendency of individual's consistent characteristics (not necessarily meaning that individual characteristics cannot be changed) in terms of thinking, remembering, processing information and solving problems (Riding, Glass & Douglas, 1993; Heineman, 1995; Sudia, 2013). From this notion of cognitive style, we can see that the cognitive and problem-solving styles are related. Therefore, in problem-solving teaching, it is necessary to consider students' cognitive style (Sudia, *et al.*, 2014).

A number of cognitive styles have been identified in the literature. For example, Abdurrahman (Sudia, *et al.*, 2014) claims that the cognitive style dimensions which gained the most attention in studying children with learning difficulties are impulsive cognitive style and reflective cognitive style. The child who is quick in answering problems but usually comes up with wrong answers is said to have an impulsive cognitive style, whereas the child who is slow in solving a problem but careful and tends to come with usually correct answers is said to have a reflective cognitive style (Kagan, 1965, Lucas-Stannard, 2003; Sudia, *et al.*, 2014).

It appears that impulsive cognitive and reflective cognitive styles are related to carefulness and carelessness in solving problems. To solve a problem, carefulness is very important, and very carefully chosen concepts and principles will lead to right solution. (Sudia, *et al.*, 2014). Impulsive or reflective styles affect the efficiency and problem-solving strategy behaviours of primary school children (Foshay & Kirkley, 1975; Sudia, *et al.*, 2014). This implies that impulsive cognitive styles and reflective cognitive styles make important contributions to problem solving. Therefore, in the writer's view, it is very important to further study impulsive cognitive and reflective cognitive styles, with regard to the thinking process of students in solving mathematical problems.

The impulsive cognitive style of students is the cause of problems, not only academic but also behavioural. In relation to the impulsive cognitive style of students, Goleman (2007) contends that the impulsive understanding system with major effect is emotional thinking. Further it is explained that the main feature of emotional thinking is quick but reckless in responding. Emotional thinking is much faster than rational thinking, it jumps immediately without considering what it is doing. That speed puts aside careful and analytical thinking that is the characteristic of rational thinking.

There are two important aspects in the measurement of cognitive style, namely: (1) the time students use to make decisions in solving problems; (2) mistakes made by students in answering the problem (Rozenchwajg & Corroyer, 2005). With regard to the first aspect, the measurement of the impulsive-reflective cognitive style is based on the amount of time students' use in solving problems. With regard to the second aspect, measurement is based on the number of mistakes students make when solving problems.

The objective of this study was to describe the thinking process of high school students of Senior High School 1 Raha students with impulsive cognitive style and reflective cognitive style in solving mathematical problems under the provision of scaffolding at each Polya stage.

Methods

This is an explorative study which aims at describing the thinking process of two high school students, one with cognitive impulsive style and the other with reflective cognitive style, in solving mathematical problems when given scaffolding. To obtain the data about the thinking process of the students, they were given the task or problem to solve, followed by interviews. The data on the result of the problem-solving task and the interview were combined, then described qualitatively. The data were in written, oral expressions, and descriptions of the subject under study, which were then analyzed (Sudia, 2013). In other words, this study used a qualitative, descriptive approach.

The subjects of this study were two students drawn from students with impulsive cognitive style and reflective cognitive-style students. The process of selecting a research subject began with the giving of a cognitive style test called the Matching Familiar Figure Test (MFFT), followed by the selection of two students with extremely different cognitive styles, both in terms of time and error. In addition, it also fulfilled criteria as follows: (1) the impulsive group was drawn from the students who were fastest in reply but careless and with answers that were mostly incorrect, and the reflective group was drawn from students who spent longest but were careful and gave answers that were mostly correct; (2) able to communicate opinion/way of thinking verbally or in writing; (3) the chosen subject had relatively equal

mathematical ability.

The two kinds of the instruments in this study were the main instrument and auxiliary instruments. The main instrument was the researcher himself, and the auxiliary were cognitive style test (MFFT), problem-solving tasks, and interview guidelines.

The data were analyzed using Miles and Huberman's analysis model (1994), and followed the following process: (1) data reduction, (2) data presentation and (3) drawing conclusions.

Results

Subject 1: Impulsive

The Stage of Understanding the Problem

The subject (1) read the problem several times until fully understanding it;(2) orally presented what had been understood; (3) expressed orally what had been known and asked in the problem; (4) mentioned the elements of the concept that would be used in solving the problem.

The Stage of Planning Problem-Solving

The subject: (1) thought about the way to solve the problem, but was not able to fully express what he thought, (2) was unable to fully express the concepts he would use to solve the problem; (3) was unable to provide reasons for using the concept to solve the problems.

The Stage of the Implementation of the Problem-Solving Plan

The subject (1) wrote down incomplete problem-solving steps; (2) could not make a representation of the given problem before being given a new scaffolding which then enabled him to make an image as a representation of the problem; (3) at the beginning of the implementation of the plan, had difficulty in the process problem solving, since not all concepts could be applied to solve the problem; (4) in implementing the problem-solving plan, he was in a hurry, so that the result of problem solving was wrong because he was not careful in solving the problem; (5) realized that the steps taken had been wrong because not all concepts could be applied to solve the problem.



The Stage of Re-Examine the Problem-Solving Result

The subject (1) expressed how to re-examine the problem-solving result, i.e. computationally; (2) was not sure that problem solving is correct.

Subject 2: Reflective

The Stage of Understanding the Problem

The subject (1) read the problem several times until he understood it well; (2) orally stated what had he understood and what was asked in the problem; (3) orally stated the condition for determining what is unknown or asked in the problem; (4) expressed verbally the truth about what had been known and asked in problem.

The Stage of Planning the Problem-Solving

The subject (1) pointed out the problem-solving steps clearly, although slowly but carefully, for example, in thinking about how to determine the base and height of the triangle, using trigonometric comparisons or Pythagoras' theorems; (2) indicated that the problem-solving steps taken were correct.

The Stage of the Implementation of the Problem-Solving Plan

The subject (1) explained the idea to solve the problem given; (2) wrote down the problem-solving steps exactly as he had in mind; (3) represented the problem by drawing two triangles that coincided, then determined the base and height of triangle 1 and triangle 2; (4) concluded that the land area was the sum of the two triangles; (5) pointed out that the troubleshooting steps had been correct; (6) in the implementation of the problem-solving plan, the subject was slow but careful.

The Stage of Re-Examining Problem-Solving Result

The subject (1) pointed out the way to re-examine the results of the problem solving; (2) re-checked the results of the problem solving and showed that it had been based on the way pointed out earlier; (3) state the truth of the result obtained of problem solving; (4) concluded that the results of the problem solving should be checked again to ensure that the results obtained had been correct.

Discussion of Findings

Subject 1: Impulsive

At the stage of understanding the problem, the impulsive subject could express in his own sentences what he had known and what had been asked in the problem, and indicated what he had not known or what had been questioned. He could also verbally express the truth of the known and what had been asked in the problem, and could mention the elements of the concepts to be used in solving the problem. This means that the impulsive subject tended to think conceptually in understanding the problem.

At the stage of planning the problem-solving, the impulsive subject considered how to solve the problem, but was unable to point out completely what he had planned in mind to solve the problem. The concept expressed to solve the problem was incomplete. Once given scaffolding, the concept to be used was complete. At the stage of planning problem-solving, the impulsive subject seemed to be in a hurry that what he was thinking was incomplete. This is in accordance with the characteristics of impulsive students, i.e. when given the problem he seemed to rush but not carefully, hence tended to be wrong (Kagan, 1965). This indicated that the impulsive subject tended to think semi-conceptually at the stage of planning problem solving.

At the stage of problem-solving plan implementation, the impulsive subject wrote the problem-solving steps though incompletely. The impulsive subject could not make a representation of the problem given, both before and after being provided with scaffolding. The impulsive subject could make the image correctly as a representation of the problem. The impulsive subject could not only memorize the formula, but can also apply the formula to solve the problem. This is in line with the Faradillah's (2014) claim, who contends that in learning mathematics students not only memorize mathematical formulas, but can also use math to solve problems in their everyday life. At the time of solving the problem, the impulsive subject seemed to be in rush, but carelessly, so he was less precise in doing the calculations. As a result, the subject made a mistake. Goleman (2007) pointed out that for the impulsive understanding system, what is greatly influential is emotional thinking. He further argues that the main feature of emotional thinking was quick but reckless in responding. The emotional thinking is much faster than the rational one, jumping immediately without considering what it is doing. The speed disregards careful and analytical thought that is the characteristic of rational thinking. The impulsive subject could not apply all concepts needed to solve the problem. Neither could he explain exactly what had been written nor that the problem-solving result had been correct. He tended to think semi-conceptually when implementing problem-solving plans.

In the stage of re-examining the problem-solving result, the impulsive subject not only mentioned the way to re-examine the result, but also re-examined the problem-solving result according to that way, i.e. computationally. However, he was not sure that the result or solution had been correct. When reviewing the problem-solving results, impulsive subjects tended to think computationally. Therefore, based on the discussion in order for impulsive students to obtain correct results in solving mathematical problems, they needed a lot of scaffolding.

Subject 2: Reflective

At the stage of understanding the problem, the reflective subject read the problem several times until he understood it properly. He then pointed out what he knew about and was asked in the problem, mentioned the conditions for determining what was unknown or asked in the problem, and expressed verbally the truth of the things known about and asked in the problem. He tended to think conceptually in understanding the problem.

At the stage of planning the problem-solving, the reflective subject wrote the precise steps he had in mind to solve the problem, in accordance with the mathematical concepts to be used in solving the problem. He was slow indeed, but he thought carefully, for instance, in thinking about how to determine the base and height of a triangle, using a trigonometric comparison or Pythagoras' theorem. He also pointed out that the steps had been correct. Indeed, it is the characteristics of reflective students that they answer problems slowly and carefully, and that they make only small mistakes (Sudia, *et al.*, 2014). In planning a problem-solving, the reflective subject was not provided with scaffolding. Therefore, it can be said that reflective subjects tend to think conceptually when planning problem-solving.

At the stage of implementing the problem-solving plan, the reflective subject expressed the idea to solve the problem, wrote down precisely the problem-solving step he already had in mind. He drew two coincided triangles to represent the problem, determined the bases and heights of the two triangles then concluded that the sum of the width of the two triangles was the width of land. He also stated that the problem-solving step is correct. In implementing the problem-solving plan, he was slow and careful; hence a correct solution. It is the characteristic of reflective students that in solving problems they are slow and careful to ensure that their answers tend to be correct (Kagan, 1965; Lucas-Stannard, 2003). When implementing a problem-solving plan, the reflective subject used concepts suitable for solving the problem given and completed it without scaffolding. This shows that when implementing a problem-solving plan, the reflective subject tended to think conceptually.

At The stage of re-examining problem-solving results, the reflective subject stated the way to re-examine the result, and re-examined the result according to the way stated. He also re-



applied the concepts used in the implementation of the problem-solving plan, and stated the truth of the result obtained. He finally concluded that the result should be re-checked to ensure it had been correct. This indicates that the reflective subject tended to think conceptually when reviewing the problem-solving result.

Conclusions and Suggestions

Based on the results of the analysis and discussion, the following conclusions can be made: (1) at the stage of understanding the problem, both subjects tended to think conceptually, but only the impulsive subject, who was provided with scaffolding, could understand the problem well; (2) at the stage of planning problem-solving, where only the impulsive subject was provided with scaffolding, the impulsive subject's thinking type tended to be semi-conceptual, whereas the reflective subject's tended to be conceptual, and; (3) at the stage of the implementation of a problem-solving plan, with regard to thinking type, the impulsive type tended to be semi-conceptual and was in such a hurry to solve the problem, that the solution tended to be incorrect, so he would be able to solve the problem correctly when provided with scaffolding; while reflective type tended to think conceptually and solved problems correctly without scaffolding; (4) at the stage of re-examining the results, the impulsive type tended to think computationally while the reflective one tended to think conceptually.

On the basis of the conclusion made, this study suggests that: (1) cognitive type tests are important for schools, especially class groupings, students' selection for Science Olympiad Competitions and other scientific competitions, even for learners' groupings both in-school learning and out-of-school learning; (2) in assigning students to a class, it is suggested that schools group students with similar students with scaffolding when learning, because it is very important to assist students, especially the one with an impulsive cognitive style, when their thinking process halts at certain stages in solving a problem.

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