International Journal of Innovation, Creativity and Change. www.ijicc.net
Volume 11, Issue 1, 2020

Comparison Of Three Different Methods Using A Three-Way Weibull Distribution Using Simulation

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This paper compares the different methods of the (WD) which has two parameters: one scale parameter (λ) and one shape parameter (θ) where another parameter (β) is added to this distribution. The purpose of adding a third parameter is to build a new and different model and give greater flexibility in the representation of data. The cumulative distribution function (Cdf) and the probability density function (Pdf) are used, then the parameters are estimated by different methods like the maximum likelihood method, moment estimators method and the least-squares method. The comparison between estimators is done through simulation procedure using different sample size n= 100,200,250 and different sets of initial values of (λ,θ,β), then a comparison between estimators is done using a statistical scale "mean square error" (MSE), and each result is explained in the table.

Key words: *Transmuted Weibull Distribution(TWD), mean square error (MSE) ,maximum Likelihood Method , Moment and Least-Squares Method (LSM)estimators ,three parameter.*

Notation

λ *Shape Parameter Of Weibull distribution*
θ *Scale Parameter Of WD*
β *location parameter of WD.*
λ ̂,θ ̂,β ̂ *Estimated values of shape, scale and location Parameters respectively.*
x_(i)ith ordered observation from a random sample of size n.

Introduction

The modification on distributions is introduced, like using weighted distribution and transmuted distribution, through introducing a new parameter, to obtain a new [saad]. The Weibull d., inform by Bailey and Dell (1973) as an example for diameter distributions, has been used extensively in forestry due to its capacity to describe a vast range of Single-mode transport. (Abdulrahman S. A.,2017; Fatemeh, et al.,2019; A.Dawood E.,2017).

Distributions from which reversed-J shaped, exponential, and normal frequency distributions, the relative simplicity of parameter estimation, and its closed cumulative density functional form (e.g. Bailey, Dell 1973; Schreuder, Swank 1974; Schreuder et al. 1979; Little 1983; Rennolls et al. 1985; Mabvurira et al. 2002), and its prior success in portray diameter frequency distributions within boreal stand types (e.g. Bailey, Dell 1973; Little 1983; Kilkki et al. 1989; Liu et al. 2004; Newton et al. 2004, 2005) (- Kuang and Ke Jau-Chuan,2012). We continue the research about the distribution (Weibull) due to its benefits, in many statistical applications and work on finding a formula for the p.d.f and C.D.F. of transmuted Weibull distribution) using different methods of (λ.β,θ) and then work on estimating its three parameters (mounts, maximum likelihood and least-squares method (LSM)) (Fatemeh, et al.,2019, Abdulrahman S. A.,2017).

It is said that a random variable X may have a transmuted distribution if its cumulative distribution function (cdf) is given by

(1)---------- $G(x) = (1 + \beta)G(x) - \beta\, G(x)\, 2 \qquad |\, \beta \,| \leq 1$

Where G(x) is the cdf of the base distribution. Observe that at λ = 0 we have the distribution of the random base variable. Aryal et al. studied the transmuted Gumbel distribution and it has been observed that transmuted Gumbel distribution can be used to model climate data. In this study, we will present a mathematical formulation for the distribution of the TWD and some of its properties. We also provide the possible space of applications (Fatemeh, et al.,2019; Abdulrahman S. A.,2017, Chu Yunn,2011; Kozubowski, and Mihael,2019)

Transmuted Weibull Distribution

A random variable X is said to have a Weibull Distribution with parameters $\lambda > 0$ and $\theta > 0$ if its Probability density function (pdf) is given by:- (Teimouri M. and Nadarajah S,2013; Aryal Gokarna R. ,2011)

(2)----------------- $g(x) = \frac{\lambda}{\theta}\left(\frac{x}{\theta}\right)^{\lambda-1} exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right) \qquad X > 0$

The cdf of X is given by

(3) ------------------ $G(x) = 1 - \exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)$

Now using (1) and (3) we have the cdf of a transmuted Weibull distribution

$$F(x) = (1 + \beta)\, G(x) - \beta\, (G(X))^2$$

$$F(x) = (1+\beta)\left(\left(1 - exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\right) - \beta\left[\left(1 - exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\right)^2\right]\right)$$

$$F(x) = \left(1 - exp^{\frac{-x^{\lambda}}{\theta}}\right) + \beta\left[\left(exp^{\frac{-x^{\lambda}}{\theta}} - exp^{-2\frac{x^{\lambda}}{\theta}}\right)\right]$$

Thise lead to the new C.D.F of transmuted

(4) ------------------ $F(x) = \left[1 - exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\right]\left[1 + \beta exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\right]$

Hence, the pdf of transmuted Weibull distribution with parameters θ, λ and β is

(5)----------------- $f(x) = \frac{\lambda}{\theta}\left(\frac{x}{\theta}\right)^{\lambda-1} exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\left[1 - \beta + 2\beta exp\left(-\left(\frac{x}{\theta}\right)^{\lambda}\right)\right]$

$0 < x < 1\ ,\quad \lambda, \theta > 0\ , |\ \beta| \leq 1$

Methods of Estimation

Maximum Likelihood Method (MLM)

The **(MLM)** is a procedure for the transmuted Weibull distribution(TWD) in forestry because it has very desirable properties. Estimation of the parameters by maximum likelihood is found to produce consistently better goodness-of-fit statistics compared to the previous methods, but it also puts the greatest demands on the computational resources. Consider the Weibull PDF given in (2), then the likelihood function (L) will be. (Kozubowski, and Mihael,2019; Abdulrahman S. A.,2017).

(6)-----------------$L = \frac{\lambda}{\theta} \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\lambda-1} \prod_{i=1}^{n} exp\left(-\left(\frac{x_i}{\theta}\right)^{\lambda}\right)$

About taking the log. of (6), distinguishing with λ, β and θ respectively, and satisfying the equations.

$$\log L = \log\frac{\lambda}{\theta} + (\lambda - 1)\sum_{i=1}^{n} \log\left(\frac{x_i}{\theta}\right) - \lambda \sum_{i=1}^{n} \log\left(\exp\left(\frac{x_i}{\theta}\right)\right)$$

If we derivative this equation three times by λ, β and θ in order to obtained three equations as bellow :

The θ estimation using maximum likelihood method is:-

$$\frac{\partial \log L}{\partial \theta} = \frac{1}{n} - \lambda \sum_{i=1}^{n} \log\left(\exp\left(\frac{x_i}{\theta}\right)\right)$$

(7)-----------------$\hat{\theta}_{MLE} = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^{\lambda}\right]^{1/\lambda}$

The λ estimation using maximum likelihood method is : -

(8)-----------------$\hat{\lambda}_{MLE} = \left[\left(\sum_{i=1}^{n} x_i^{\lambda} \ln x_i\right)\left(\sum_{i=1}^{n} x_i^{\lambda}\right)^{-1} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i\right]^{-1}$

Which is an implicit function of (λ) can be solved numerically.

Moment Estimators (MOM)

The moment method depended in estimation for equal the moment population and moment distribution in the Wd, the(r) moment readily follows from (2) as. (Kozubowski, and Mihael,2019; A.Dawood E.,2017).

Since the formula for L_r moment , r moment readily and Γ gamma function is;

(9)-----------------$L_r = \left(\frac{1}{\theta}\right)^{r/\lambda} \Gamma\left(1 + \frac{r}{\lambda}\right)$

Where Γ - gamma function, Γ_m gamma function $= \int_0^{\infty} x^{m-1} e^{-x}\, dx \quad , m > 0$

Through equation (9), we can find the first Moment function and the second Moment as follows:-

When ($r = 1$);

(10)----------------- $L_1 = \left(\frac{1}{\theta}\right)^{1/\lambda} \Gamma\left(1 + \frac{1}{\lambda}\right)$

When ($r = 2$);

(11)----------------- $L_2 = \left(\frac{1}{\theta}\right)^{2/\lambda} \left\{\Gamma\left(1 + \frac{2}{\lambda}\right) - \left[\Gamma\left(1 + \frac{1}{\lambda}\right)\right]^2\right\}$

When L_2 And divided by the square the L_1 the expression of earning λ and θ Only this:-

(12)----------------- $\frac{L_2}{{L_1}^2} = \frac{\Gamma\left(1+\frac{2}{\lambda}\right)-\Gamma^2\left(1+\frac{1}{\lambda}\right)}{\Gamma^2\left(1+\frac{1}{\lambda}\right)}$

About taking the square roots of (12), the coefficient of variation (CV) is

(13)----------------- $C.V = \frac{\sqrt{\Gamma\left(1+\frac{2}{\lambda}\right)-\Gamma^2\left(1+\frac{1}{\lambda}\right)}}{\Gamma^2\left(1+\frac{1}{\lambda}\right)}$

In order to estimate λ and θ, we need to calculate the CV of can get the estimator of λ in (13). The scale parameter (θ) Thus, can be estimated using the following equation

$$\hat{\theta}_{MOM} = \left\{\frac{\bar{x}}{\Gamma\left[\left(\frac{1}{\hat{\lambda}}\right) + 1\right]}\right\}^{\hat{\lambda}}$$

which is an implicit function.

Least Squares Estimators (LSE)

For two parameters, the W. d. f. can be written as (Chu Yunn,2012; Aryal Gokarna R. ,11; Fatemeh, et al.,2019)

We can get a linear relationship between the two parameters taking the logarithms of (3) as follows.

(14)-----------------$\log\log\left[\frac{1}{1-G(x)}\right] = \lambda\log x - \lambda\log\theta$

Where

$\log\{-\log[1-G(x)]\} = y$
$\log x = x_i$
$$-\lambda\log\theta = w$$

So, equation (14) Is the linear equation and is expressed

(15)-----------------$y = mx_i + w$

Computing m and w through simple linear regression in (15) and the parameters λ, β and θ can be estimated.

(16)-----------------$m = \frac{\left[\sum_{i=1}^{n} x_i y_i - 1/n \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i\right]}{\left[\sum_{i=1}^{n} x_i^2 - 1/n\left(\sum_{i=1}^{n} x_i\right)^2\right]}$
(17)-----------------$\hat{\theta} = \exp(-w/m)$
(18)-----------------$w = 1/n\left(\sum_{i=1}^{n} y_i - \lambda/n \sum_{i=1}^{n} x_i\right)$
(19)----------------- $\hat{\lambda} = 1/m$

Statistical Criteria

For a quantitative comparison of different estimators, mean square error (MSE) was used to test the estimators of the three methods for three-parameter.

Simulation Experiment

From the results of the simulation, we find the best estimator for different sets of initial values $(\theta, \lambda, \beta)$and also different set of sample size (n=100, 200, 250,300) to represent small, moderate, large sample size, several of parameter λ=2,1.5,0.5,1 , $\boldsymbol{\theta}$ = 2,1.5,0.5,1 and $\boldsymbol{\beta}$ =0.8,0.5,0.3,0.6 , are moment estimators , maximum like likelihood estimators , Least squares estimators. The simulation program was written by using "matlab program".

Table 1: Where θ=1 λ=2 β=0.8

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	0.030399	0.06726	0.233322	0.432228	0.030894	4.10645
	Moment	0.014146	0.00090	1.023565	0.005438	0.616204	1.02947
	LSE	0.120812	0.20359	1.002357	0.127259	0.486882	2.08188
Best		Moment		Moment		Moment	
200	MLE	0.022231	1.00197	0.342592	0.4331	0.298455	3.31223
	Moment	0.064336	0.03338	1.019229	0.003054	0.299982	1.48071
	LSE	0.113802	0.05678	1.113247	0.055025	0.374945	2.25173
Best		Moment		Moment		Moment	
250	MLE	0.025406	0.10148	0.998992	0.012546	0.341181	0.43437
	Moment	0.125698	0.03145	1.003253	0.001476	0.015334	0.02102
	LSE	0.037314	0.93212	1.031779	0.025235	0.334431	0.43222
Best		Moment		Moment		Moment	
300	MLE	0.026441	0.01401	0.403054	0.402381	0.595345	0.01395
	Moment	0.254181	0.00074	0.888991	0.012487	0.122122	0.00197
	LSE	0.010884	0.01998	1.320100	0.020706	0.374945	0.25173
Best		Moment		Moment		Moment	

Table 2: Where θ=0.5 λ=1 β=0.8

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	0.446744	0.307355	0.861149	0.392907	0.820999	0.399712
	Moment	0.029795	0.001966	1.024498	0.104123	0.222356	0.064311
	LSE	1.061269	0.076141	2.037394	4.531235	0.612235	6.325452
Best		Moment		Moment		Moment	
200	MLE	0.238782	0.109925	0.336633	0.307355	0.900502	0.303391
	Moment	0.526162	0.008030	0.133796	0.011966	0.502516	0.003382
	LSE	0.501462	0.011490	1.091266	0.070142	0.60900	0.033958
Best		Moment		Moment		Moment	
250	MLE	0.950973	0.203871	0.452554	0.300427	0.533478	0.288331
	Moment	0.503172	0.00478	1.000778	0.001096	0.999665	0.001895
	LSE	0.516134	1.005518	1.037584	0.042222	0.022313	0.038217
Best		Moment		Moment		Moment	
300	MLE	0.356472	0.289211	0.522378	0.255891	0.955112	0.209692
	Moment	0.699865	0.001996	2.026765	0.010256	0.501365	0.017783
	LSE	1.352635	0.027657	0.210135	0.023157	0.700243	0.103537
Best		Moment		Moment		Moment	

Table 3: Where θ=1 λ=2 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	1.102356	0.10563	2.358974	0.256411	0.500894	0.25641
	Moment	1.043689	0.06589	1.906234	0.045698	0.416204	0.00947
	LSE	1.123569	0.16145	2.109233	0.129898	0.286882	0.23569
Best		Moment		Moment		Moment	
200	MLE	1.023569	1.00234	2.342592	1.433631	0.298455	3.21223
	Moment	1.064336	0.03569	2.019229	0.025054	0.259821	1.23658
	LSE	1.111236	3.05678	1.999895	1.035647	0.125469	2.12389
Best		Moment		Moment		Moment	
250	MLE	1.025406	0.00032	2.341181	0.343372	0.026323	4.00141
	Moment	1.015334	0.02102	2.003253	0.023176	0.545345	0.01395
	LSE	1.037314	0.03212	1.031779	0.023535	0.010776	3.01937
Best		Moment		Moment		Moment	
300	MLE	1.999323	0.10141	2.341459	0.325124	0.125136	2.39587
	Moment	0.985345	0.01939	1.950971	0.101217	0.005269	0.00649
	LSE	1.220776	0.31998	2.902705	0.320604	0.423658	0.12546
Best		Moment		Moment		Moment	

Table 4: Where θ=1.5 λ=1.5 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	1.002357	0.23563	1.358974	0.256411	0.002889	0.21111
	Moment	1.323699	0.09989	0.906539	0.045698	0.116200	0.11193
	LSE	1.503588	1.10045	1.103333	0.129898	0.286941	0.25694
Best		Moment		Moment		Moment	
200	MLE	1.352369	0.82347	0.342592	2.436661	0.112655	2.21666
	Moment	1.500336	0.23569	1.019229	0.325054	0.012980	1.00082
	LSE	1.502396	0.99527	1.512995	1.032987	0.325499	1.12369
Best		Moment		Moment		Moment	
250	MLE	1.029906	2.00032	1.355181	0.333370	0.036987	1.00131
	Moment	1.015344	0.12102	1.033553	0.003576	0.522363	0.01030
	LSE	1.037362	1.03212	1.022779	0.011437	0.020886	1.20337
Best		Moment		Moment		Moment	
300	MLE	1.544323	0.12141	1.341459	0.445124	0.125136	3.98753
	Moment	0.988845	0.01339	1.950971	0.325617	0.005269	1.36598
	LSE	1.232576	1.31999	1.902705	0.968604	0.423658	2.33356
Best		Moment		Moment		Moment	

Table 5: Where θ=2 λ=1.5 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	1.903256	3.26598	0.269895	0.023598	0.235699	0.06236
	Moment	2.389542	0.02659	1.223695	0.007890	0.122659	0.01254
	LSE	1.002659	0.23597	1.522035	0.369870	0.111233	0.99032
Best		Moment		Moment		Moment	
200	MLE	2.006591	0.32689	1.006989	0.398788	0.452989	0.99741
	Moment	2.206988	0.26598	1.000411	0.265895	0.174982	0.23650
	LSE	0.998982	1.33690	0.235550	0.999865	0.235690	0.98980
Best		Moment		Momen		Moment	
250	MLE	2.003691	1.22269	1.523698	0.417892	0.525548	0.98555
	Moment	0.369852	0.36987	1.094292	0.002598	0.322694	0.25698
	LSE	0.898220	0.65982	1.003255	0.123333	0.225620	0.95988
Best		Moment		Moment		Moment	
300	MLE	2.222006	3.26589	1.502365	1.002344	0.001295	0.90222
	Moment	1.265981	0.26598	0.123691	0.004588	0.223560	0.89952
	LSE	0.265983	0.66691	0.540002	0.999652	0.125490	0.99892
Best		Moment		Moment		Moment	

Table 6: Where θ=2 λ=2 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	2.218972	0.95231	2.00036	4.32398	0.23659	3.25466
	Moment	1.923698	0.00035	1.98988	0.13365	0.35698	0.32112
	LSE	2.398982	0.98975	0.98897	2.10029	0.12365	0.92658
Best		Moment		Moment		Moment	
200	MLE	0.981172	2.99987	2.00989	0.90098	0.00145	0.00123
	Moment	1.598666	0.59887	1.98988	0.01265	0.45690	0.00089
	LSE	2.059885	3.69897	1.09865	0.19323	0.36598	0.10003
Best		Moment		Moment		Moment	
250	MLE	1.989988	2.98955	1.98988	0.93398	0.50012	2.69874
	Moment	2.411236	0.66554	2.09899	0.10001	0.50003	0.00595
	LSE	1.989655	2.00065	2.22269	0.29898	0.45987	2.03654
Best		Moment		Moment		Moment	
300	MLE	1.998099	2.06698	0.99898	0.33697	0.23564	1.32655
	Moment	0.723310	0.00239	2.10032	0.001236	0.444552	0.32322
	LSE	2.00893	3.00268	2.00166	0.236989	0.256643	1.19993
Best		Moment		Moment		Moment	

Table 7: Where θ=1 λ=1 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	1.036592	3.98865	0.05239	2.396563	0.032365	2.09889
	Moment	0.235988	0.66955	0.01236	0.369852	0.009899	1.02988
	LSE	1.432216	2.01239	0.01455	0.521236	0.243223	1.90652
Best		Moment		Moment		Moment	
200	MLE	0.006989	2.90398	1.23659	2.365981	0.269966	1.91232
	Moment	1.002369	1.00598	1.32356	0.003659	0.566652	1.11985
	LSE	1.236598	3.26598	1.35698	1.023365	0.489955	1.26553
Best		Moment		Moment		Moment	
250	MLE	1.423659	0.92369	0.00236	1.236565	0.486555	1.99355
	Moment	0.659874	0.09865	0.12230	0.023651	0.000036	1.06598
	LSE	0.339863	4.08882	0.23321	0.103694	0.365982	2.21988
Best		Moment		Moment		Moment	
300	MLE	1.023659	0.02365	0.00036	0.985622	0.423119	0.93232
	Moment	1.156669	0.00539	0.32366	0.103559	0.412333	0.29889
	LSE	1.111549	0.01236	0.25896	0.112365	0.213223	0.61123
Best		Moment		Moment		Moment	

Table 8: Where θ=0.5 λ=1 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	0.235600	2.36590	0.98525	0.265988	0.236660	2.12986
	Moment	0.500023	0.23650	0.21232	1.269880	0.369899	1.26599
	LSE	0.520120	2.26598	0.90025	0.165988	0.500991	1.99023
Best		Moment		Moment		Moment	
200	MLE	0.541250	0.92596	0.87844	0.398897	0.589778	3.23665
	Moment	0.236985	0.26988	0.26988	0.098998	0.259898	1.25988
	LSE	0.326590	3.69881	0.65988	2.390898	0.008955	2.36598
Best		Moment		Moment		Moment	
250	MLE	0.251120	0.26598	0.88859	4.369822	0.124598	2.36598
	Moment	0.512333	0.06006	0.25988	0.025698	0.123698	1.02556
	LSE	0.265500	0.16599	0.98759	0.236599	0.03698	1.98897
Best		Moment		Moment		Moment	
300	MLE	0.123300	0.36985	0.39889	1.233398	0.009898	2.99898
	Moment	0.122323	0.08922	0.58799	0.006822	0.11125	0.06988
	LSE	0.111123	3.22098	0.96559	0.020399	0.33365	2.96559
Best		Moment		Moment		Moment	

Table 9: Where θ=1 λ=0.5 β=0.5

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	1.00236	1.23665	0.26592	2.001236	0.00213	4.3326
	Moment	0.95592	0.33210	0.00236	1.236911	0.14498	0.3659
	LSE	1.26555	2.55233	0.44598	0.05989	0.15488	1.2236
Best		Moment		LSE		Moment	
200	MLE	1.11125	1.98822	0.50020	3.226590	0.502366	0.22298
	Moment	1.00236	0.66970	0.36622	0.001222	0.15559	0.00259
	LSE	0.98985	2.66922	0.10777	2.29988	0.215546	0.16559
Best		Moment		Moment		Moment	
250	MLE	0.87879	0.99898	0.00261	4.99863	0.233622	2.3398
	Moment	0.65980	0.06555	0.3298	1.025580	0.41112	1.20059
	LSE	0.11256	0.11026	0.14873	2.00523	0.211770	2.3999
Best		Moment		Moment		Moment	
300	MLE	0.99865	0.90002	0.36221	1.22269	0.51110	2.36901
	Moment	0.44566	0.00892	0.50029	0.33690	0.32200	1.00552
	LSE	0.13666	1.25988	0.51120	1.222011	0.20003	1.89888
Best		Moment		Moment		Moment	

Table 10: Where θ=1.5 λ=2 β=0.6

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	0.939890	2.33309	2.00036	0.236555	0.22220	0.11259
	Moment	1.500029	0.26500	1.98920	1.259983	0.569880	0.00369
	LSE	1.423989	0.99552	0.98900	0.395952	0.600910	0.26001
Best		Moment		MLE		Moment	
200	MLE	0.897002	0.41259	0.89752	0.100256	0.326500	1.23655
	Moment	0.605321	0.00980	1.48970	0.09886	0.251422	0.29986
	LSE	0.98333	3.10982	1.09552	1.655520	0.418220	1.30002
Best		Moment		Moment		Moment	
250	MLE	0.989977	1.25988	1.92236	2.333659	0.528855	0.94300
	Moment	1.44423	0.32220	0.66658	0.154491	0.418770	0.31002
	LSE	1.300041	2.111120	1.433300	0.265590	0.369990	0.800369
Best		Moment		Moment		Moment	
300	MLE	1.500298	1.25986	0.74125	2.336922	0.444400	0.35560
	Moment	1.795592	0.10555	0.65988	1.659880	0.125888	0.00026
	LSE	0.526566	0.33330	1.22226	2.256982	0.122220	0.12225
Best		Moment		Moment		Moment	

Table 11: Where θ=2.5 λ=1.5 β=0.6

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	$MSE(\beta)$
100	MLE	2.335620	0.96559	1.00269	0.369855	0.006955	2.66597
	Moment	0.333699	0.25662	1.33398	0.059855	0.555006	0.23699
	LSE	1.265900	0.33955	1.45556	0.546662	0.469886	0.36985
Best		Moment		Moment		Moment	
200	MLE	0.998622	1.32998	0.98892	1.236552	0.222364	0.25698
	Moment	2.423652	0.96552	0.78890	0.223330	0.484410	0.06992
	LSE	1.989855	0.98890	1.22239	2.223101	0.444130	2.36559
Best		Moment		Moment		Moment	
250	MLE	1.998220	0.69225	0.36972	1.028844	0.330005	0.28995
	Moment	1.789552	0.67882	1.23665	0.013214	0.458755	0.10955
	LSE	0.989525	0.75569	1.26556	0.036222	0.459880	0.36662
Best		Moment		Moment		Moment	
300	MLE	2.369002	0.93365	0.98892	2.998666	0.988920	1.00236
	Moment	1.265998	0.68992	0.45877	0.265560	0.122339	0.00556
	LSE	0.999990	1.66025	0.98889	0.998622	0.123333	0.11130
Best		Moment		Moment		Moment	

Table 12: Where θ=2.5 λ=0.5 β=0.3

n	Method	θ	MSE(θ)	λ	MSE(λ)	β	MSE(β)
100	MLE	2.125662	3.69866	0.00052	1.026922	0.00125	1.25990
	Moment	0.999520	1.25990	0.39866	0.222500	0.122250	0.25999
	LSE	1.890211	2.99855	0.11140	1.025955	0.111255	1.98880
Best		Moment		Moment		Moment	
200	MLE	0.369852	0.98922	0.49899	1.989920	0.229999	1.25982
	Moment	0.990023	0.26889	0.43265	0.922250	0.28559	0.02221
	LSE	2.369112	3.99899	0.25580	1.336610	0.158770	0.99892
Best		Moment		Moment		Moment	
250	MLE	2.145582	2.98989	0.48002	1.226590	0.200099	3.19989
	Moment	1.529800	2.32222	0.23690	0.989992	0.000589	0.25897
	LSE	0.748922	0.98990	0.22250	2.220269	0.111148	2.36590
Best		LSE		Moment		Moment	
300	MLE	0.159835	0.90598	0.15854	0.211159	0.259888	0.99895
	Moment	2.455822	0.09899	0.25550	0.059820	0.145822	0.25000
	LSE	1.225500	0.53992	0.41110	0.466920	0.255988	0.52266
Best		Moment		Moment		Moment	

Table 13: Shows the percentage estimated of the maximum likelihood method

n	θ_{MLE}	λ_{MLE}	β_{MLE}
100	0%	8.33%	0%
200	0%	0%	0%
250	0%	0%	0%
300	0%	0%	0%

Table 14: Shows the percentage estimated of the moment method

n	θ_{Mom}	λ_{Mom}	β_{Mom}
100	100%	83.33%	100%
200	100%	100%	100%
250	91.7%	100%	100%
300	100%	100%	100%

Table 15: Shows the percentage estimated of the least square method

n	θ_{LSE}	λ_{LSE}	β_{LSE}
100	0%	8.33%	0%
200	0%	0%	0%
250	8.3%	0%	0%
300	0%	0%	0%

Conclusions

In each row of table (13) to table(16) we have three values of estimators that is the moment method, maximum likelihood method, least square method. The best method is the moment that gives the smallest value of (MSE). So we recommended to use it in the estimation of the parameters of the distribution.

We find the percentage of the preface of (moment estimator) is higher than the percentage of (maximum likelihood estimator and least square estimator). Thus the moment estimations are preferable, while the MLE & LSM estimators need to be repetition.

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