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# Middle School Students' Different Conceptions of the Equal Sign: Case study of Indonesian Students

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This paper reveals results from a written assessment given to 168 students in grade seven at middle schools in Pekanbaru, Indonesia. The authors discuss the students' response when given equation problems addressing their interpretation of the equal sign. The authors find that there are four kinds of misinterpretation in solving equation problems. The evidence of this study led to students' views in the operational rather than the relational aspect. Although students have begun to learn introductory algebra, they are usually doing calculations from left to right side, as operational, rather than doing structural relations. This also indicates that the students havea lack of understanding of the meaning of the equal sign. Furthermore, it is a base for the students to learn algebra and a vital contribution to early algebra experiences in middle school.

**Key words**: *Equation, Equal Sign, Middle School Students, Operational, Relational*

## Introduction

Researchers in mathematics education generally agree that algebra is essentially unifying ideas in mathematics. Algebra is often described as a gateway to further mathematics learning. Additionally, algebra has historically served as a gatekeeper to advanced mathematics and higher education (Asquith et al., 2007, Cai et al., 2005) (Jupri et al., 2014, McNeil et al., 2006) (Stephens et al., 2013). In short, algebra is a tool for solving mathematics (Kooij, 2001); it also provides a language that mathematics teaches (Stacey and Chick, 2004).

National Council of Teachers of Mathematics revealed that algebra is for all ages (NCTM, 2000). Algebra is identified as a significant strand of the curriculum for students from grade K-12 (Brizuela et al., 2015, Carraher et al., 2006) (Greenes, 2002). Moreover, algebra is still

required to analyse and solve the problems in advanced mathematics and higher education. Algebra is even used for an initial stage in qualifying testing to be accepted at work.

Some educational researchers, curriculum developers, and policymakers began to explore experiences that students in elementary and middle schools might need when they face algebra (Kaput et al., 2017, Dougherty et al., 2015). One recently popular topic is about how to develop the students' primary thinking about algebra. It is not for formal algebra but for how to think algebraically.

In algebra, there is an object to indicate our action in solving the problem. One of the accessible objects is the signs which are commonly used in algebra, such as; =, >, <, ≥, ≤, or ≠ (Arcavi et al., 2016). These signs are also used in arithmetic; however, they are in arithmetic and algebra having a different role. In elementary schools, the equal sign generally appears in a non-algebraic sense, and it assigns a result to the operations. The equal sign mostly appears as for the result of the operations, for example, the number sentence such as 5 + 6 = 11. The operational view generally teaches the students, such 5 + 6 = 11, they seldom led the form of equation 5 + 6 = 4 + 7. The conceptions of the students with the equal sign as an operational signifier, signalling, for example, the answer, the sum is equal to; bring the students an operational view (Asquith et al., 2007, Stephens et al., 2013) (Vermeulen and Meyer, 2017). On the other hand, the equal sign indicates the equivalence between two terms (Schwarzkopf et al., 2018, Jupri et al., 2014) in which the values on both sides of the equal sign are sameness.

When the students solve the equation problem both in the numerical and algebraic task, the understanding of the relational conception of the equal sign is a vital aspect which the student begins in doing algebraic manipulations. The equal sign is a crucial algebraic concept. This concept must be encountered and understood by the students beginning in the lower grades (Knuth et al., 2008, Stephens, 2006). Additionally, equal sign in an essential algebra concept means that student must maintain the facade and be encouraged to understand in the early grades (Chesney and McNeil, 2014, Falkner et al., 1999) (Hohensee, 2017, Kilpatrick et al., 2001) (Knuth et al., 2008). This concept is about the relationship between two quantities; that each side is equivalent. Students are prerequisite to understanding that the equal sign is the equivalence of the two things.

Furthermore, understanding the equal sign is an underlying discussion in early algebra. Kieran (2004) indicated that students should refocus on the meaning of the equal sign. The limited conception about the meaning of the equal sign is one of the critical hesitant blocks in learning algebra (Knuth et al., 2008). Virtually all manipulation on algebra problem requires the equal sign to represent a relation. Furthermore, Kieran (2004) also expressed that the notion of equivalence is one of the possible adjustment for students to make a smooth transition from arithmetic to algebraic thinking.

Research has exposed that many students from elementary school and later years have an operational view rather than a relational view about the equal signs (Kieran, 1981, Knuth et al., 2008) (Stephens, 2006, Stephens et al., 2013) (Vermeulen and Meyer, 2017). Students with an operational concept view the equal sign as "put the answer", come to the answer, or "do something signalled" (Kieran, 1981, Prediger, 2010) (Stephens et al., 2013). Afterwards, students with a relational concept view the equal sign as "the sameness", "what is on the left and right of the sign mean the same thing", "the same as" (Behr et al., 1980, Knuth et al., 2008) (Stephens et al., 2013). A wide-range understanding of the equal sign from a relational concept brings the vital notation in solving equations in that the sign is as the numerical or algebraic problem and giving the lies of the heart of algebraic manipulation problems.

In this article, the authors present how middle school students interpret their answering and understanding of the meaning of the equal sign, which was presented in the equation form. In Indonesia elementary schools, the equal sign generally appears in a non-algebraic sense, and it assigns a result to the operations. Meanwhile, in secondary schools, the role of the equal sign indicates the equivalence between two terms. Therefore, the students are mostly thinking to solve the equation problem operationally.

Moreover, in Indonesian curricula, the meaning of the equal sign is not explicitly contained in the document. Therefore, the teachers are not aware of the role and meaning of the equal sign. Wahyuni dan Herman (2019) mentioned that Indonesian material textbook is still giving the students a formal situation rather than the idea for developing algebraic thinking. Hence, the students still perceive and solve algebra problem as operational, and it is caused they have not an abstraction meaning about the equal sign as equivalence while it is the basic notion of learning algebra.

As an international issue, this topic is familiar, particularly in developing algebraic thinking for making a smooth transition for students from arithmetic to algebraic thinking. The authors chose this topic since this topic should be identified in the early year, and the equal sign is seldom getting an interpretation from the students' perspective, especially for Indonesian student's cases. Not much research in Indonesian focuses on views about the equal sign case. The formulated the research problem is what are characteristics of the misinterpretation view of the Indonesian students about the meaning of the equal sign.

**Students' Understanding of Equal Sign**

Symbols are commonly used in algebra. The most widely used mathematics symbol is the equal sign. All levels of schools use this symbol to solve math problems. Understanding these symbols is mainly a vital idea for students in learning algebra. The symbol of the equal sign also represents the students' understanding before learning algebra. Many elementary and middle school students have misinterpreted the understanding of the meaning of the equal

sign. The duality function of the equal sign gives a different interpretation of the meaning of the equal sign.

Students often perceive the equal sign as the result of an arithmetic operation rather than as a symbol of mathematical equivalence (Falkner et al., 1999, Kieran, 1981) (Knuth et al., 2006, Vermeulen and Meyer, 2017). The students tend not to view the equal sign as the relationship of the equivalence of two sides, but they view it as a signal to perform computations from left to right side. When they are given to the problem of the forms p + q = …, the students automatically perceive it as a stimulus for the answer to be placed in the blank (Johanning, 2004, Matthews et al., 2012) (Stephens et al., 2013). Meanwhile, a sentence like 8 + 4 = __ + 5 will be perceived by the students as a stimulus to do something rather than to express a relationship of equivalence.

The equal sign is defined as a relational symbol, primarily, if there is a relationship between two sides of an equation (Powell, 2015). This relationship is often interpreted as "the same as" in expressing a relation between two equivalent expressions. For students who have a relational view, the the equal sign is demonstrated more flexibly when working with equations in a non-traditional format such as 15 = 5 +10, 8 = 8 or 8 + 4 = __+ 5 (Stephens et al., 2013). The students will have flexibility in overcoming equation problems if they already have the relational view. The flexibility of the students will initiate their understanding of the structure of algebra in the form of abstract ideas. Hence, students who do not view the equal sign as relational are less successful in solving an algebra problem, in term equation, rather than the students who understand the equal sign as a relation between two sides (Knuth et al., 2006).

## Methods

### Participants

Data were collected from 168 seventh grade students at two middle public schools and one private middle school in Pekanbaru, Indonesia. The students had learned a whole number as the first topic and algebra as the second topic. The school has a regular mathematics curriculum in Indonesia called 2013 Curriculum revision 2017. In this research, the author intended to examine and analyse the student's misinterpretation. The assessment was given to the students, and it led the students' answer based on the previous research (Asquith et al., 2007, Falkner et al., 1999) (Knuth et al., 2006, Matthews et al., 2010) (Stephens et al., 2013).

### Data Collection

The data were collected from the students' response to a subset of items from a multiple-choice test (Sesli and Kara, 2012, Lau et al., 2011). These responses were the initial evidence

concerning the students' understanding of the meaning of the equal sign. This test was administered at the beginning of the middle school year to identify the students' understanding of arithmetic and structure that led to learning algebra. To support the data, the authors interviewed the students. It aimed to obtain more deep information from the students.

**Data Analysis**

To gather efficient data and accurate information, the author displayed the coding of the items. Students' responses to the item were responses such as "the answer comes next" while the students expressed the idea of "put answer after the symbols" or "add the numbers". While the students' responses by using all the numbers in the question, the student expressed the idea "add all the numbers" or "everything the numbers in the equation we added". These two statements will be coded as the operational view, which means the students' interpretation of the meaning of the equal sign.

**Results and Discussion**

The research found that many middle school students had an operational view of the equal sign (Knuth et al., 2008, McNeil et al., 2006). Similarly, the authors also find that it also occurs to the middle school students in Pekanbaru in solving the equation problems. First, the authors identify the students' answer and analyse the students' misinterpretation about the equal sign. Then, the authors focus on the students' strategy in solving the equation.
Moreover, there are four different misinterpretations of the students that are observed in this research:

a. As the last process of the operation (the answer comes next)

When students are solving the equation 8 + 4 = __ + 5, the number to replace __ is 12 (Falkner et al., 1999, Stephens et al., 2013). It is similar to the finding of this research, in which the students endeavour to solve 45 + 12 = Δ + 14. The results of this research indicate that 76 students responded by writing 57 to replace Δ in the equation. Based on figure 4, there are 46% of students took 57 as the answer. This result indicates that most students contributed wrong answers and nearly half of the total students occupied this in their answer sheet. Most of the students think as operationally. Therefore, it is necessary to explore how the students interpret that 57 is the correct answer. To know the students get the answer, the interviews are conducted to several students, for example, AD, to find out their interpretations of the problem in the equation.

*R : Have ever had questions like this?*
*AD : No ma'am*
*R : Is it difficult to solve this problem?*

*AD : No ma'am*
*R : Can you solve this problem?*
*AD : (Students return to solve the problem and stop solving problems*
*R : Are you sure about the answer?*
*AD : yes ma'am*
*R : how to solve this problem?*
*AD : Add it (point the number 45 + 12), ma'am, 45 + 12 = 57, now there is an answer, 57*
*R : Are there any other reasons?*
*AD : no ma'am*
*R : oke, I have another one for you 3 + 4 = 2 + 5, true or false?*
*AD : (think a long time), true*
*R : how do you get it?*
*AD : well, 3 + 4 = 7 and 2 + 5 = 7, so it is true. (he remember his answering previously) If this is true, so the previously false the answering 57 (he rethink his answering)*

Based on the interview, the student expresses that this problem is not difficult for him. Additionally, he never solves the problem similar to their question. However, based on the results of the answer, AD writes the wrong answer, which was 57. The repetition of the students solving the problem from left to right side that indicate from his statement "add the numbers". The student only notices the addition operation so that he adds 45 + 12 without looking at other numbers. He is not considering the other number, and he just thinks that operationally the numbers.

As a consequence, he only sees that there is a sum of 45 and 12 not the equality of 45 + 12 and Δ + 14. On the other hand, while the students give another number, one digit, he rethinks about their thinking previously. He takes a long time for these students to announce true. His habit of doing operational cause conflict in his mind while comparing another number. The student is not considering the problems that will exist in his daily learning, both numerical problem. The misinterpretation of this student, it will interfere with the students while solving $ax + b = cx + d$ (Blanton et al., 2015, Johanning, 2004) (Linchevski and Livneh, 1999).

Lastly, this case indicates we conclude that students only put the answer as the end of the addition operation. The results given by these students can be seen on the answer sheet (see figure 1).

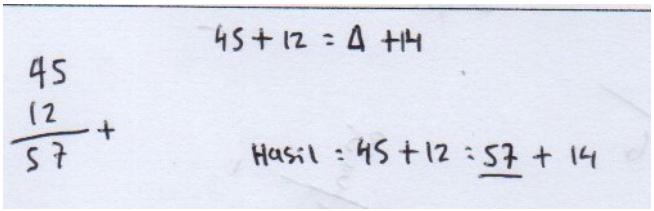


**Figure 1. Misinterpretation of the student that placed 57 as the answer**

Thus, it can be concluded that the student's view towards the equal sign is the operational understanding. It is because the student put the answer after the symbol.

b. Use all the numbers in the equation

This research finds different results of the answers given by the student (AL). This student expresses to solve the equation by adding all the numbers in the equation and put the answer in the triangle (see Figure 2).

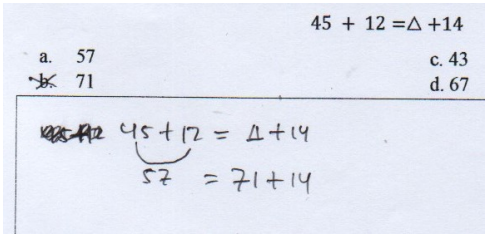


**Figure 2. Misinterpretation of the student that placed 71 as the answer**

AL writes the sum of 45 + 12 that is 57 and adds 14; therefore, he gets 71 as the answer. The unique thing is that the student writes the answer on the triangle which replaced number. Student only looks at three numbers in the equation and additional operations. He adds all the numbers, and the number to replace the triangle is 71.

c. The students extended the problem to the incorrect “equation string.”

According to Stephens et al., (2013) another response to the equal sign is that the students might reveal a different interpretation about the equal sign. At this point, the equation 45 + 12 = Δ + 14 were solved by the students by writing 71 to replace Δ as the final action in the right. These students, like their interpretation that 45 + 12 = 57, still add 14 then to complete the action their give additional the equal sign. Hence, he adds 14 to 57; then, he gets the answer 71 (see Figure 3). Most of the students choose the answering 71, mostly give this reason and misinterpretation.

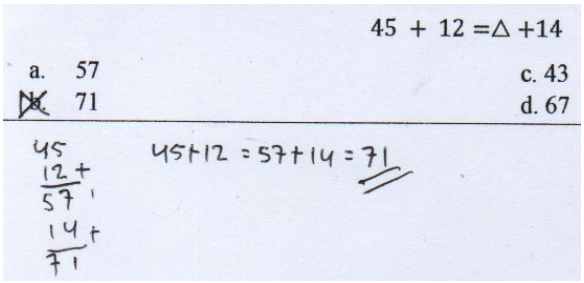


**Figure 3 Misinterpretation of the students that extended the equal sign**

d. The students expressed by the term number

In this case, the students' strategy is by seeing the side term of numbers. The student considers the side term in left and right. An additional case in completing an equation is that the student carries out an addition operation by looking at the numbers separately. This case means that the student sees the sides of numbers in the right 12 and another right 14 which are both located to the right of the statement then the student adds up the numbers 12 and 14. from that operational, and he gets the result 26 and then it seems not found in options. Later, the student proceeds by adding 45 to get an answer 71.

Based on the results, there are four misinterpretations of the student while solving the equation. For answering 71, there are three strategies on how to get the answer 71. Although it is the wrong answer, it is interesting to discuss their interpretation. Mostly their answering of 71 is caused by the habitual student's way of thinking about numbers. While the students face a lot of the numbers, the students will do so operationally.

Overall, the authors indicating that all of the students' answers in equation 45 + 12 = Δ + 14 are mostly incorrect. It is affected that the students also got misinterpretation about the equation. A few students completed the equation correctly. Based on figure 4, our finding that there are four options provided for students, only three answers are chosen by the students. The data demonstrate that no student chose to answer 67, and the higher students take the answer 57. This finding approves that students mostly have an operational view rather than a relational view. It is supported by figure 4 that the students who answered 43 are fewer than students who answered 57. The diagram can be seen in Figure 5.

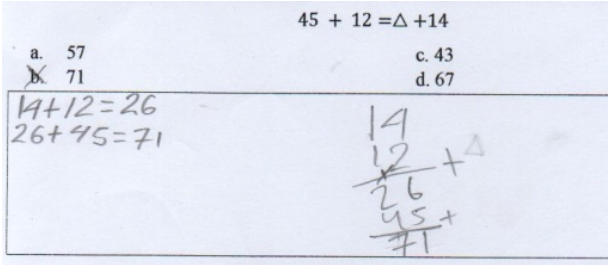


**Figure 4 Misinterpretation of the students that added by the term of numbers**

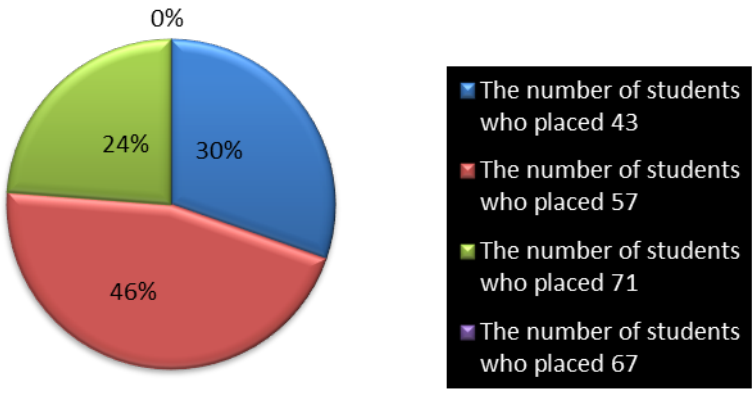


**Figure 5. The number of students who solved the equation 45 + 12 = Δ + 14**

Another finding that the students' operational view is still much influenced by their habits at the elementary school. It seems from the student stated that he is not facing this equation previously. Then in another issue that in the Indonesian elementary textbook, students rarely led to finding the meaning the equal sign. In arithmetic, students are most concerned with calculating the answer. In fact, while non-standard numerical problems are given for them, they are spontaneously doing the calculation—this case of Indonesian students showing that. Our finding provided proof with the understanding of the Indonesian students while they are facing the nonstandard equation for interpreted the meaning of the equal sign and equivalence.

## Conclusions

In this paper, the researchers have investigated the students' misinterpretation of the meaning of the equal sign. The authors in this research clarify the meaning of the equal sign for Indonesian students in which the implementation of the equal sign is not included in the curriculum from the developing country such as Indonesia. The case of Indonesian students gives the four misinterpretations about the meaning of the equal sign. The case of student thinking of operational and number, giving the student think of different meaning. The topic is still interesting to explore and gain for further research. However, this study still has some limited items and data for gathering information about the whole misinterpretation from the students. Besides, this is a multiple-choice assessment in which the exploration of the items was needed to get comprehensive information about the students' interpretation and to obtain the vital idea for the students before they learn algebra.

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